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Design of slender reinforced concrete rectangular columns subjected to eccentric loads by approximate methods

Projeto de pilares retangulares esbeltos de concreto armado submetidos à flexão composta reta por métodos aproximados



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Abstract

Reinforced concrete codes worldwide establish that the design of slender columns must ensure that under the most unfavorable load combination, there is neither instability nor material failures. Thus, it is mandatory to consider material as well as geometrical nonlinearities. The consideration of second order effects can be done using simplified methods or the general method. This work analyses second order effects based on the approximate methods shown in NBR 6118 [1]: approximate curvature method and approximate stiffness procedure. Due to the importance of the columns in the stability of buildings is essential that these simplified design methods provide safe solutions for the design of columns. In this scenario, the objective of this study is to evaluate these simplified design procedures in terms of safety, precision and economy with respect to test results of RC slender columns subjected to eccentric loads found in the literature. The comparative analysis reveals that the approximate stiffness procedure provides better results.

Keywords: reinforced concrete slender columns, eccentric loads, second order effects, simplified design methods

Resumo

Normas para projeto de estruturas de concreto armado estabelecem que a análise dos efeitos de 2ª ordem em pilares esbeltos deve ser feita de modo a assegurar que para as combinações mais desfavoráveis das ações de cálculo, não ocorra perda de estabilidade, nem esgotamento da capacidade resistente de cálculo. Para isso é obrigatório considerar a não-linearidade física dos materiais, juntamente com a não-linearidade geométrica. Os cálculos dos efeitos de 2ª ordem podem ser feitos por métodos aproximados ou pelo método geral. Neste trabalho o foco será no cálculo desses efeitos de 2ª ordem com base nos métodos aproximados prescritos na NBR 6118: método do pilar-padrão com curvatura aproximada e método do pilar-padrão com rigidez aproximada. Dada a importância dos pilares e de sua estabilidade é importante que os métodos aproximados sejam capazes de apresentar soluções seguras para o dimensionamento destes. Neste cenário, o objetivo desse trabalho é avaliar estes métodos aproximados em termos de segurança, precisão e economia com relação a resultados de ensaios de pilares encontrados na literatura. A análise deste estudo comparativo revela que o emprego do método da rigidez aproximada apresenta resultados mais próximos dos encontrados nos ensaios de pilares.

Palavras-chave: pilares esbeltos de concreto armado, cargas excêntricas, efeitos de segunda ordem, métodos aproximados de cálculo.

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1. Introduction

Reinforced concrete (RC) columns are linear structural elements, usually cast vertically, responsible for carrying the loads from floors to the foundations. The stability of any given structure is directly linked to the stiffness and strength of the columns. Thus, the design of reinforced concrete columns must include local checks as well as global analysis of the structural system.
Concrete codes worldwide require that columns must be designed to resist not only axial and bending moments computed from an ordinary first order frame analysis including allowances for construction imperfections but also moments due to internal force effects resulting from deflections (second-order effects). Thus the design of concrete columns must be based on the factored forces and moments from a second-order analysis considering material nonlinearity and cracking, as well as the effects of member curvature and lateral drift, duration of the loads, shrinkage and creep, and interaction with the supporting foundation. Column sections, under the above conditions, must then be designed to ensure that there is neither instability nor material failures.
Use of such an analysis to determine column compressive axial forces and bending moments for section design is the most rational approach. But this analysis is very complex. Thus concrete codes worldwide allow the use of approximate design methods for slender columns. The simplified methods shown in NBR 6118 [1] are *approximate curvature method and approximate stiffness procedure*. Due to the responsibility of columns in the stability and strength of concrete structures, these approximate approaches must provide adequate safety in the design.
In this scenario, the goal of this paper is to analyze NBR 6118 [1] approximate design criteria for slender rectangular columns subjected to eccentric loads with respect to safety, precision and economy, by comparing code based calculations with respect to experimental results of columns built with conventional concrete ($f_c \leq 55$ MPa). The investigation also includes columns with concrete having compressive strength above 55 MPa since NBR 6118 procedures are being changed to allow the use of concrete with compressive strength above 55 MPa.

2. Methodology

2.1 NBR 6118 approximate design approaches for slender columns

The NBR 6118 [1] provides two simplified design procedures for the evaluation of second order effects in slender columns: approximate curvature method and approximate stiffness procedure. The simplifications in both procedures are related to geometric and material nonlinearities.

2.1.1 Approximate curvature method

The approximate curvature method is applicable to RC columns with slenderness ratios less or equal to 90 and symmetrical location of the reinforcement. It can only be used in columns subjected to axial loads and bending on one axis. The geometrically nonlinear behavior is simplified by assuming a deformed shape represented by a sine curve. The material nonlinearity is taken into account by an approximate equation for the curvature at the critical column cross-section.
The lateral displacement e_2 due to the second order effects is given by:

$$e_2 = \frac{\ell_e^2}{10} \cdot \frac{1}{r} \qquad (1)$$

The curvature is calculated from:

$$\left(\frac{1}{r}\right) = \frac{0{,}005}{h(\nu + 0{,}5)} \leq \frac{0{,}005}{h} \qquad (2)$$

with

$$\nu = \frac{N_d}{A_c \cdot f_{cd}} \geq 0{,}5 \qquad (3)$$

The column maximum bending moment is then equal to:

$$M_{d,tot} = \alpha_b M_{1d,A} + N_d \cdot e_2 \quad with\ \alpha_b M_{1d,A} \geq N_d\,(1{,}5 + 0{,}03h) \qquad (4)$$

These symbols used are explained in the notation.

2.1.2 Approximate stiffness procedure

The approximate stiffness procedure is applicable to RC columns subjected to combined flexure and axial loads with unsupported length to radius of gyration ratio less or equal to 90 and symmetrical location of the reinforcement. It can only be used in columns with rectangular cross-sections. The deformed shape in this case is also represented by a sine curve. The material nonlinearity is taken into account by an approximate equation for column flexural stiffness.
The maximum design bending moment in a column is equal to:

$$M_{d,tot} = \frac{\alpha_b . M_{1d,A}}{1 - \dfrac{\lambda^2}{120\, k/\nu}} \geq M_{1d,A} \qquad (5)$$

where the dimensionless stiffness $\boldsymbol{k}$ is calculated from:

$$k = 32 . \nu \left(1 + 5 . \frac{M_{d,tot}}{h . N_d}\right) \qquad (6)$$

The symbols used are also explained in the notation.
The dimensionless stiffness κ is needed to calculate $M_{d,tot}$ and is also a function of $M_{d,tot}$. Thus an iterative process must be used. However, Scadelai [2] has shown that an iterative process is not necessary and $M_{d,tot}$ can be calculated from:

$$M_{d,tot} = \frac{5.M_1 - k_2 + \sqrt{k_2^2 + 10.M_1\left(2.h.N_d - k_2\right) + 25.M_1^2}}{10} \geq M_{1d,A} \quad (7)$$

with

$$M_1 = \alpha_b \; M_{1d,A} \quad (8)$$

$$k_1 = 1 - \frac{\lambda^2}{3840} \quad (9)$$

$$k_2 = k_1 \; h \; N_d \quad (10)$$

2.2 Slender reinforced concrete columns database

The database for this analysis was originally assembled by Souza [3] as part of his final undergraduate report. It consists of slender columns tested up to failure in laboratories worldwide. All columns have longitudinal and transversal (stirrups) reinforcement, and were tested in one axis bending with axial compressive loads. Columns fabricated with concrete containing light aggregates or fibers are not included.
The most important characteristics of the columns in the databank are shown in Tables 1 and 2. Each table corresponds to a range of concrete compressive strength: up to 55 MPa and above 55 MPa. Each table also presents the number of columns tested by each research team, their respective geometrical characteristics, the longitudinal reinforcement ratio as well as the concrete compressive strength. The indicated concrete compressive strength were obtained from 15 cm x 30 cm cylinder specimens.

2.3 Comparative Study

The comparative study between the actual and predicted second order effects can be quantified based on the ratio of the test failure moment M_{test} to the predicted one. The test failure moment M_{test} provides a close estimate of the true capacity. It can be determined from the loads and the total lateral displacements measured at the column failure. The design moment M_{pred} is calculated using equations 4 and 7 respectively. For the evaluation of the predicted moment M_{pred}, all material resistance factors were set equal to one. Further, the measured concrete compressive strength f_c of each test specimen was used in determining M_{pred}.
For each column the ratio M_{test}/M_{pred} was calculated. Statistical analyses of this ratio include its average ***m***, the median m_d, the standard deviation ***SD***, the coefficient of variation ***CV*** as well as the maximum and minimum values. The average of M_{test}/M_{pred} is used as a measure of the conservative bias of the procedure while the coefficient of variation is taken as an indication of accuracy.
With the objective of evaluating the reliability and of comparing the performance of shear design code equations for reinforced concrete beams, COLLINS [4] developed a demerit point scale methodology. Considering safety, precision and economy, a score is attributed for each range of M_{test}/M_{pred} ratio: these values are

Table 1 – Column database ($f_c \leq$ 55 MPa)

Research team	Number of tested columns	b (cm)	h (cm)	l_e (cm)	λ	f_c (MPa)	ρ_l (%)
ENCISO (5)	4	25	15	312	72	46,9 to 53,6	1,30 to 4,30
ADORNO (6)	6	25	12	215	62	36,9 to 42,5	1,05
LEE and SON (7)	6	21	12	138 - 210	40 - 61	34,9 to 41,8	1,13
DANTAS (8)	5	25	12	300	86,7	34 to 38	1,57
LIMA JÚNIOR (9)	3	15	15	195	45	39,2	2,18
SANTOS (10)	11	25	12	200 - 250	58 - 72	37,8 to 45,8	1,57
MELO (11)	4	25	12	300	86,7	39,6	1,57
GALANO and VIGNOLI (12)	6	10	10	212	73,4	43,1	2,01 to 4,52
CLEASON and GYLLTOFT (13)	6	12 - 20	12 - 20	260 - 420	55 - 75	33 to 43	2,1 to 3,2

Table 2 – Column database (f_c > 55 MPa)

Research team	Number of tested columns	b (cm)	h (cm)	l_e (cm)	λ	f_c (MPa)	ρ_l (%)
LEE and SON (7)	14	12	12	138 - 210	40 - 61	70,4 to 93,2	1,98 to 5,51
LIMA JÚNIOR (9)	3	15	15	195	45	66,4	2,18
LLOYD and RANGAN (14)	18	30	10	168	58	58 to 97,2	1,51 to 2,26
CLEASON and GYLLTOFT (13)	6	12 - 20	12 - 20	260 - 420	55 - 75	86 to 93	2,1 to 3,2
GALANO and VIGNOLI (12)	24	10	10	212	73,4	75,2 to 113,3	2,01 to 4,52

shown in Table 3. A score value smaller than 0,5 is worse in terms of safety than one greater than 2. On the other hand, a score value equal 2, corresponded to the low safety zone is attributed to an extremely conservative ratio for being not economical. The total demerit point score of each design code equation is calculated by summing the products of the percentage of M_{test}/M_{pred} obtained in each range times the demerit value attributed to that range. The largest the total demerit point score is the worst is the performance of the design equation.

3 Results and analysis

3.1 Columns made with concrete of $f_c \leq 55$ MPa

The results of the M_{test}/M_{pred} ratio for columns made with concrete of $f_c \leq 55$ MPa are presented in the Table 4. They were obtained from the comparison with test results of 51 columns. The analysis of these results reveals that NBR 6118 [1] approximate stiffness criterion is more conservative than the approximate curvature method: average and median values are larger than the unity. In terms of accuracy, the approximate stiffness procedure is also more precise, since the coefficient of variation is 2% smaller. With respect to safety, the demerit scale (shown in the part B of Table 4) reveals that the results obtained with the approximate curvature method are of concern: 22% of the M_{test}/M_{pred} values are below the appropriate safety range (smaller than 0.85) with 2% in the extremely dangerous zone (smaller than 0.5). This fact

Table 3 – Demerit Point Classification of Collins (4)

Classification	M_{test}/M_{pred}	Score
Extremely dangerous	< 0.50	10
Dangerous	0.50 |---- 0.65	5
Low Safety	0.65 |---- 0.85	2
Appropriate Safety	0.85 |---- 1.30	0
Conservative	1.30 |---- 2.00	1
Extremely Conservative	⩾ 2.00	2

Table 4 – Columns made with concrete of $f_c \leqslant$ 55MPa

Part A – Statistical Analysis

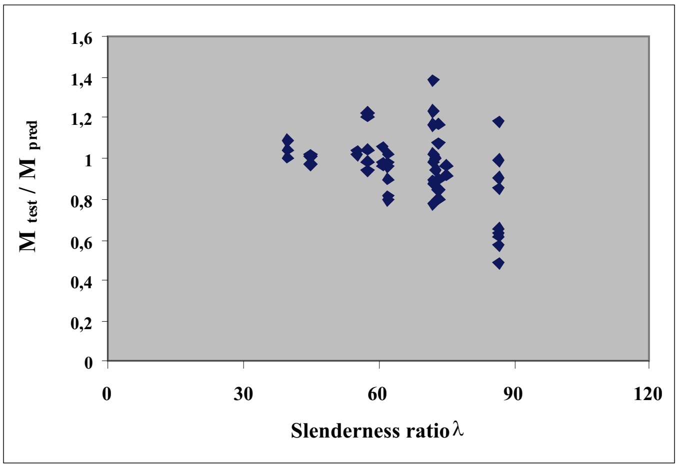
Parameters	M_{test}/M_{pred} Approximate Curvature Method	M_{test}/M_{pred} Approximate Stiffness Procedure
Average (m)	0,962	1,051
Median (m_d)	0,978	1,073
Standard deviation (SD)	0,178	0,173
Coefficient of variation (CV)	18,55 %	16,43 %

Part B – Demerit Point Classification

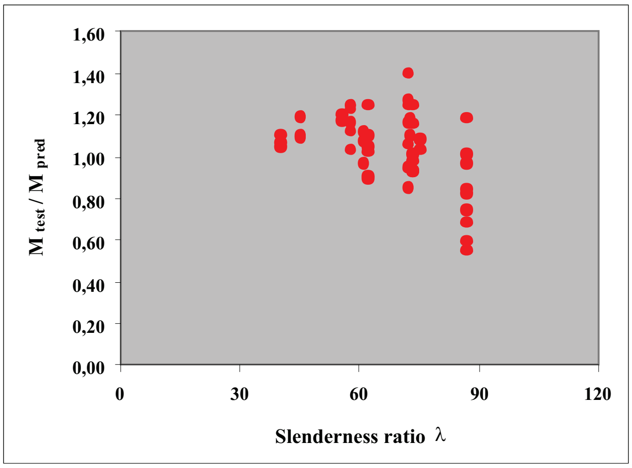
M_{test}/M_{pred}	Approximate Curvature Method	Approximate Stiffness Procedure
< 0.50	2*	0
0.50 |--0.65	5,9	3,9
0.65 |--0.85	13,7	7,8
0.85 |--1.30	76,5	86,3
1.30 |--2.00	2	2
⩾ 2.00	0	0
Total Demerit Point Score	**79****	**37**

* – Percentage values of M_{test}/M_{pred} results
**79 = (2 x 10) + (5,9 x 5) + (13,7 x 2) + (76,5 x 0) + (2 x 1) + (0 x 2)

Figure 1 – M_{test}/M_{pred} *versus* column slenderness ratio λ



A Approximate curvature method



B Approximate stiffness procedure

Table 5 – Columns made with concrete of f_c > 55MPa

Part A – Statistical Analysis		
Parameters	M_{test}/M_{pred} Approximate Curvature Method	M_{test}/M_{pred} Approximate Stiffness Procedure
Average (m)	1,026	1,146
Median (m_d)	1,012	1,125
Standard deviation (SD)	0,184	0,173
Coefficient of variation (CV)	17,94%	15,09%
Part B – Demerit Point Classification		
M_{test}/M_{pred}	Approximate Curvature Method	Approximate Stiffness Procedure
< 0.50	0*	0
0.50 | --0.65	3,1	0
0.65 | --0.85	12,3	4,6
0.85 | --1.30	78,5	81,5
1.30 | --2.00	6,2	13,9
⩾ 2.00	0	0
Total Demerit Point Score	**46****	**23**

* – Percentage values of M_{test}/M_{pred} results
** 46 = (0 x 10) + (3,1 x 5) + (12,3 x 2) + (78,5 x 0) + (6,2 x 1) + (0 x 2)

is also reflected in the total demerit point score of this method, which is much larger. The approximate stiffness procedure, on the other hand, has smaller percentage of results in the low safety and dangerous range with no values in the extremely dangerous zone.
The effects of concrete compressive strength f_c, of the column slenderness ratio λ and of the dimensionless axial force υ on the ratio M_{test}/M_{pred} were verified for both the approximate curvature method and the approximate stiffness procedure. For both design methods, this analysis revealed a reduction in the M_{test}/M_{pred} ratio for larger values of the column slenderness λ as shown in Figure 1. For the other two parameters no trend was found.
Overall, the analysis shows that NBR 6118 [1] approximate stiffness criterion provides better predicting results in terms of safety, precision and economy for columns made with concrete of $f_c \leq 55$ *MPa* .

3.2 Columns made with concrete of f_c > 55 MPa

Since NBR 6118 [1] procedures are being changed to allow the use of concrete with compressive strength above 55 MPa, a comparative analysis of the current approximate design approaches for slender columns is presented next.
The results of the M_{test}/M_{pred} ratio for columns made with concrete of $f_c > 55$ *MPa* are presented in the Table 5. They were obtained from the comparison with test results of 65 columns.
The analysis of these results reveals that NBR 6118 [1] approximate stiffness criterion and approximate curvature method have average and median values for the M_{test}/M_{pred} ratio larger than the unity. The approximate stiffness criterion is more conservative since its average and median are larger. In terms of accuracy, the approximate stiffness procedure is also more precise, since the coefficient of variation is 3% smaller. With respect to safety, the demerit scale (shown in the part B of Table 5) shows that the results obtained with the approximate stiffness criterion are safer with only 4,6% of the M_{test}/M_{pred} values below the appropriate safety range (smaller than 0.85) and all of them are in the low safety range. This fact is also reflected in the total demerit point score of this method, which is much smaller. The approximate curvature method, on the other hand, has larger percentage of results in the low safety and dangerous range with no values in the extremely dangerous zone.

Analysis of the effects of concrete compressive strength f_c, of the column slenderness ratio λ and of the dimensionless axial force υ on the ratio M_{test}/M_{pred} were also done in this case for both approximate design procedures. For both design methods, the results (Figure 2) showed a slight reduction in the M_{test}/M_{pred} ratio with increasing values of the dimensionless axial force υ. For the other two parameters no trend was found.
The analysis of these results also indicates that NBR 6118 [1] approximate stiffness criterion provides better predicting results in terms of safety, precision and economy for columns made with concrete of f_c **> 55 MPa**.

4. Concluding remarks

The goal of this paper was to analyze with respect to safety, precision and economy NBR 6118 [1] approximate design criteria for slender rectangular RC columns subjected to eccentric loads by comparing to experimental results. The study includes columns built with conventional concrete ($f_c \leq 55$ MPa) and with concrete having compressive strength above 55 MPa. The simplified methods prescribed by NBR 6118 [1] analyzed herein were - *approximate curvature method and approximate stiffness procedure*. For the comparative study a database was created containing the results of columns tested in the laboratory.
The overall analysis of the comparative study indicates that the approximate stiffness procedure provides better results than the approximate curvature method in relation to the experimental results of columns built with conventional concrete ($f_c \leq 55$ MPa) and fabricated with concrete having compressive strength above 55 MPa.

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6. Notation

A_c = gross area of column cross-section.
b = side dimension of column cross section.
f_c = concrete cylindrical compressive strength at time of tests.
f_{cd} = design concrete compressive strength.
h = overall column height.
l_e = column unsupported length.
$M_{d,tot}$ = total bending moment in midheight section.
$M_{1d,A}$ = column maximum factored end moment.
N_d = factored axial force.
a_b =factor relating actual moment diagram to an equivalent uniform moment diagram.
λ = slenderness ratio.
ν = dimensionless axial force.
ρ_λ =longitudinal steel ratio.

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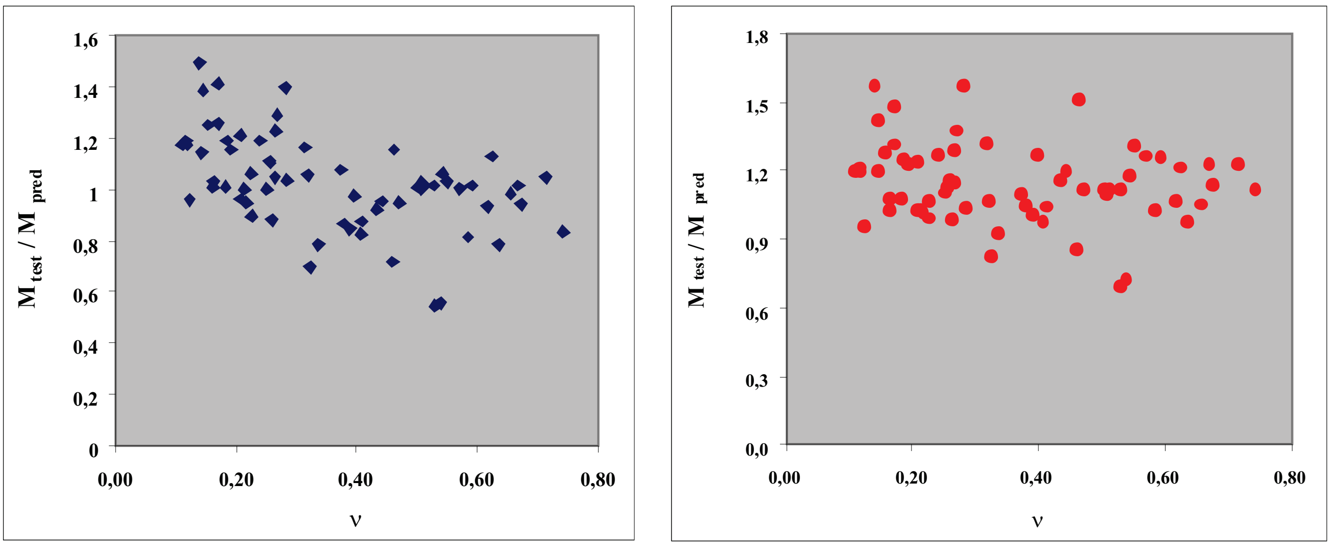
Figure 2 – M_{test}/M_{pred} *versus* dimensionless axial force ν



A Approximate curvature method
B Approximate stiffness procedure

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