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# Efficiency evaluation of Branson's equivalent inertia to consider physical nonlinearity of beams in simple form

# *Avaliação da eficiência da inércia equivalente de Branson para considerar a não-linearidade física das vigas de forma simplificada*

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## Abstract

In this paper the efficiency of Branson's equivalent inertia to consider the physical nonlinearity of beams in simple form is evaluated. For this purpose, several reinforced concrete plane frames of medium height are analyzed using ANSYS software. Initially, the frames are processed considering both physical and geometric nonlinearities. Next, geometric nonlinear analyses are performed, considering a physical nonlinearity approximated through the stiffness reduction in the structural elements. In the case of the columns, the stiffness was reduced by 20% and, for the beams, the inertia reduction values according to the Branson [1] formula and the NBR 6118:2007 [2] Brazilian Norm were used. It was observed that the inertia reduction according to the Branson [1] formula better represents the actual behavior of the structures at the service limit state. Furthermore, it was verified that the use of Branson's equivalent inertia is more efficient at representing the behavior of the more flexible frames than stiffer frames.

***Keywords***: reinforced concrete, physical nonlinearity, Branson's equivalent inertia.

## Resumo

Neste trabalho busca-se avaliar a eficiência da inércia equivalente de Branson [1] para considerar a não-linearidade física das vigas de forma simplificada. Com este objetivo, são realizadas diversas análises numéricas de pórticos planos medianamente altos em concreto armado utilizando o "software" ANSYS. Inicialmente, os pórticos são processados considerando ambas as não-linearidades geométrica e física. Em seguida, são realizadas análises não-lineares geométricas, considerando a não-linearidade física de forma aproximada, por meio da redução de rigidez dos elementos estruturais. No caso dos pilares, a rigidez foi reduzida em 20% e, para as vigas, foram utilizados os valores de redução de inércia segundo a formulação de Branson [1] e aqueles recomendados pela NBR 6118:2007 [2]. Observa-se que a redução de inércia segundo a formulação de Branson [1] representa o comportamento das estruturas com maior precisão no estado limite de serviço. Além disso, mostra-se que a utilização da inércia equivalente de Branson [1] é mais eficiente para representar o comportamento dos pórticos mais flexíveis do que dos pórticos mais rígidos.

***Palavras-chave:*** concreto armado, não-linearidade física, inércia equivalente de Branson.

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## 1. Introduction

In recent decades, following the example of other areas, engineering has made great advances, particularly regarding designs and civil construction. Optimization techniques related to terms of weight and form, the development of testing equipment and computers and efficient numerical modeling have lead to more economical and elegant constructions and to higher buildings and bolder designs.
Therefore, issues not properly approached before have come to be of fundamental importance in structural design. Of note among these issues is stability analysis and second order effect assessment.
Second order effects occur when the equilibrium analysis for the structure is conducted based on its deformed configuration. As a result, the existing forces interact with the displacements, producing additional efforts. The second order efforts introduced by horizontal displacements lead to joints in the structure, when subjected to vertical and horizontal loads, are called second order global effects; these effects may be extremely important and significant in some structures; in others, they need not be taken into account.
In stiffer structures, the horizontal displacements of the joints are small and, consequently, the second order global effects have a small influence on the total efforts, and may then be dispersed. These structures are called nonsway structures.
On the other hand, there are more flexible structures, the horizontal displacements of which are significant and for which, therefore, the second order global effects represent an important part of the final efforts, as they cannot be dispersed; this is the case with sway structures.
According to NBR 6118:2007 [2], if the second order global effects are less than 10% of the respective first order efforts, the structure may be classified as a nonsway structure. In this case the efforts obtained through the first order analysis are applied. However, if the second order global effects are in excess of 10% of the first order effects, the structure is classified as a sway structure and must be analyzed taking into consideration the effects of the geometric and physical nonlinearities.
It is clear, therefore, that analysis of a sway structure is much more complex than that for a nonsway structure. This is because conducting an analysis that takes into consideration the effects of geometric and physical nonlinearities, for reinforced concrete structures, may result in an arduous task, demanding great amounts of computing power and tools that are not always available in the offices where the calculations are made. It therefore becomes essential that simplified methods of simulating, safely, the effects of geometric and physical nonlinearities on the structure be developed.
The consideration of geometric nonlinearity demands more refined analyses, which take into consideration some degree of modification to the structure stiffness matrix, or the utilization of simplified processes, such as the final efforts assessment method (which include second order ones) employing the global instability coefficient $\gamma_z$ as a magnifier of the horizontal loads.
Taking the physical nonlinearity in consideration requires determining the stiffness of each structural element based on the constitutive relationships of the materials, the quantity and the disposition of the reinforcement in the element and the level of demand it requires. As this is a labor-intensive process, many studies have been conducted that deal with physical nonlinearity in an approximate manner, by means of a reduction in the stiffness of the structural elements.
In this paper the goal is to assess the efficiency of Branson's inertia equivalent [1] to consider the physical nonlinearity of beams in simple form. With this objective, various numerical analyses are performed of reinforced concrete plane frames of medium height utilizing ANSYS software. Initially, the structures are processed considering both the geometric and physical nonlinearities. Next, geometric nonlinearity analyses are performed, dealing with physical nonlinearity in an approximate manner by means of a reduction in the stiffness of the structural elements. In the case of columns, the stiffness was reduced by 20% and, for beams, inertia reduction values were used in accordance with Branson's formula [1] and the formulas recommended in NBR 6118:2007 [2]. The results of the geometric nonlinearity analyses, with the simplified physical nonlinearity method are, then, compared with those obtained through the geometric and physical nonlinearity analyses, which are capable of representing the actual behavior of structures with greater precision.

## 2. Simplified physical nonlinearity method according to NBR 6118:2007 [2]

NBR 6118:2007 [2] establishes, for the approximation of physical nonlinearity, the following structural element stiffness values:
- slabs: $(EI)_{sec} = 0.3\, E_{ci} I_c$;
- beams: $(EI)_{sec} = 0.4\, E_{ci} I_c$ when $A'_s \neq A_s$ or $(EI)_{sec} = 0.5\, E_{ci} I_c$ when $A'_s = A_s$;
- columns: $(EI)_{sec} = 0.8\, E_{ci} I_c$;

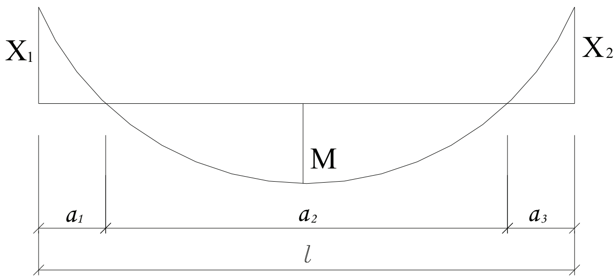
in which:
- $I_c$ – moment of inertia of gross concrete section;
- $A'_s$ – area of compression reinforcement;
- $A_s$ – area of tension reinforcement;
- $E_{ci}$ – initial concrete elasticity modulus, arrived at by:

$$E_{ci} = 5600\sqrt{f_{ck}} \qquad (MPa) \tag{1}$$

- $f_{ck}$ – characteristic strength of the concrete to compression, in MPa.

The norm also allows, when the bracing structure is composed only of beams and columns and the global instability coefficient $\gamma_z$ is less than 1.3, for the adoption of $(EI)_{sec} = 0.7\, E_{ci} I_c$ for both the elements.
It is worth mentioning that, according to the results of Pinto et al. [3], reductions in stiffness equal to *0.4 EI* and *0.5 EI* for beams and *0.8 EI* for columns have been shown to be safe, including the value of *0.4 EI* for beams for which $A'_s \neq A_s$, which is the most common situation, is even a bit low. In addition, it appears more rational to adopt different reductions in stiffness for beams and columns, since the limit state of cracking in these elements is not the same, due to the efforts to which they are subjected.
The adoption of a single value of *0.7 EI* for beams and columns was probably done in an effort to facilitate the analysis of the structure. However, according to Lima [4], this procedure must be used with caution, especially when the beams make a significant contribution to global stiffness.

Figure 1 – Obtaining of the pondered equivalent stiffness for continuous beams




## 3. Equivalent stiffness according to the Branson formula [1]

In the case of reinforced concrete beams, different amounts of reinforcement and the variable distribution of cracking along the span mean that stiffness against *EI* bending will not be constant.
According to NBR 6118:2007 [2], verification of rotations and displacements in linear structural elements must be performed using models that take into account the effective stiffness of the cross sections of the elements, taking into consideration the presence of reinforcement, the cracking of the concrete along this reinforcement and the strains over time.
Branson [1] presents an empirical expression in order to determine the effective stiffness in any particular cross section of a beam. This effective stiffness is a function of the bending moment, the section's properties and the concrete strength.
The equivalent stiffness produced by Branson's formula [1], which has been adopted by NBR 6118:2007 [2], for an approximate assessment of the immediate deflection in beams may be written as follows:

$$(EI)_{eq} = E_{cs}\left\{\left(\frac{M_r}{M_a}\right)^3 I_c + \left[1-\left(\frac{M_r}{M_a}\right)^3\right] I_{II}\right\} \leq E_{cs} I_c \quad (2)$$

where:
- $E_{cs}$ is the secant concrete elasticity modulus, arrived at using:

$$E_{cs} = 0.85\ E_{ci} \quad (3)$$

with $E_{ci}$ defined by the equation (1);
- $I_c$ is the inertia moment for the gross section of concrete;
- $I_{II}$ is the inertia moment for the cracked section of concrete in state II;
- $M_a$ is the bending moment in the critical section of the span being considered, the maximum moment in the span for beams with two or more supports and the moment in support for cantilevers, for the combination of actions considered in this assessment;
- $M_r$ is the cracking moment of the structural element, calculated using:

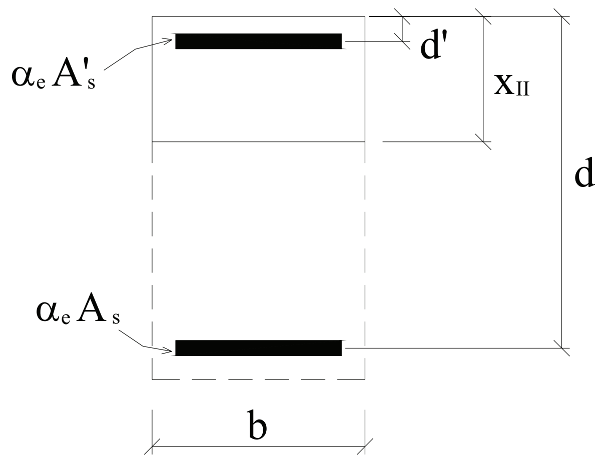
$$M_r = \frac{\alpha \cdot f_{ct} \cdot I_c}{y_t} \quad (4)$$

with $\alpha$ equal to 1.5 for rectangular sections and 1.2 for T or double-T sections, $y_t$ being the distance from the center of gravity to the most tensioned fiber, and $f_{ct}$ the strength to the direct tension of the concrete, in accordance with item 8.2.5 of NBR 6118:2007 [2].
According to the ACI Committee 435 [5], the equivalent stiffness can be obtained with greater precision, for spans with continuous beams, by pondering the equivalent stiffness values of the regions. Thus, for the spans shown in figure [1], the value pondered for the equivalent stiffness is arrived at using:

$$(EI)_{eq,pond} = [(EI)_{eq,1}.a_1 + (EI)_{eq,2}.a_2 + (EI)_{eq,3}.a_3] / l \quad (5)$$

where $(EI)_{eq,i}$ represents the equivalent stiffness of the three regions in the figure [1]. In each of the regions, the equivalent stiffness should be calculated using the equation (2), using the values $X_1$, $M$ and $X_2$, respectively, for $M_a$.
In order to determine the inertia moment for the cracked section $I_{II}$ in the equation (2), elastic and linear behavior was admitted for the steel and concrete being compressed, while disregarding the concrete tension (figure [2]).
The section should initially be homogenized, using the following relationship between the steel and concrete elasticity modulus:

Figure 2 – Cracked section (state II)



$$\alpha_e = E_s / E_{cs} \quad (6)$$

Next, the depth of the neutral axis in state II, $x_{II}$, was obtained, equalizing the static moment of the area above the neutral axis ($Q_{sup}$) with the area below ($Q_{inf}$). Thus, we have:

$$Q_{sup} = Q_{inf} \quad (7)$$

$$[(b.x_{II}).x_{II}/2 - A'_s.(x_{II} - d')] + \alpha_e.A'_s.(x_{II} - d') = \alpha_e.A_s.(d - x_{II}) \quad (8)$$

$$(b.x_{II}).x_{II}/2 + (\alpha_e - 1).A'_s.(x_{II} - d') = \alpha_e.A_s.(d - x_{II}) \quad (9)$$

Replacing

$$\alpha'_e = \alpha_e - 1 \quad (10)$$

in equation (9), the following second degree equation in $x_{II}$ is obtained:

$$(b/2).x_{II}^2 + (\alpha_e.A_s + \alpha'_e.A'_s).x_{II} - (\alpha_e.A_s.d + \alpha'_e.A'_s\, d') = 0 \quad (11)$$

which results in:

$$x_{II} = - A + (A^2 + B)^{½} \quad (12)$$

with

$$A = (\alpha_e.A_s + \alpha'_e.A'_s)/b \quad (13)$$

$$B = 2.(\alpha_e.A_s.d + \alpha'_e.A'_s\, d')/b \quad (14)$$

For the inertia moment for cracked section $I_{II}$, the result is:

$$I_{II} = (b/3).x_{II}^3 + \alpha'_e.A'_s.(x_{II} - d')^2 + \alpha_e.A_s.(d - x_{II})^2 \quad (15)$$

It is important to mention that the equation (2) should only be utilized when the bending moment $M_a$ is equal to or in excess of the cracking moment $M_r$, or that is, when $M_r/M_a \leq 1$ (state II). When $M_r/M_a > 1$, the structure is already in state I and, therefore, the stiffness of gross section $E_{cs}I_c$ must be utilized.
Supposing, for example, that $M_r/M_a = 0.5$, the equation (2) is:

$$(EI)_{eq} = E_{cs}.\{(0.5)^3.I_c + [1 - (0.5)^3].I_{II}\} \quad (16)$$

$$(EI)_{eq} = E_{cs}.(0.125.I_c + 0.875.I_{II}) \quad (17)$$

Note that, in this case, the equivalent stiffness $(EI)_{eq}$ is determined, predominantly, by the stiffness of the cracked section $(EI)_{II}$, with the contribution from the stiffness corresponding to the section of gross concrete being greatly reduced. It is common, therefore, for $M_a/M_r$ relationships in excess of 2 to adopt an approximation of $(EI)_{eq}$ equal to $(EI)_{II}$.

## 4. Numeric applications

In this paper, various plane frames belonging to regular reinforced concrete buildings (the typical stories used in these buildings can be found in Oliveira [6]) are analyzed using the finite element method utilizing ANSYS 9.0 software. Table [1] summarizes the main characteristics of the examples studied.
Initially, linear elastic analyses of the buildings were performed utilizing three-dimensional models. The load acting on the structures was divided into two groups: the vertical load and the horizontal load.
The vertical load is composed of permanent loads and of the accidental load. The permanent loads considered were the weight of the structure itself, masonry loads and the slab coatings and finishings.
The horizontal load is constituted of the loads equivalent to the action of the wind, in the directions parallel to the *X* and *Y* axes. The drag forces were calculated in accordance with the NBR 6123:1988 [7] Brazilian Norm and the ultimate normal combinations followed the rules established in NBR 6118:2007 [2].
The reinforcement of the columns and beams that constituted the frames being studied was determined based on the envelope of the efforts obtained for each load combination. The beams were dimensioned to combined axial load and bending and the columns to combined axial load and bending or to combined axial load and biaxial bending. CA-50 steel was used for all the structural elements, with an elasticity modulus equal to 210 GPa.
The plane frames were then processed considering both the geo-

**Table 1 – Main characteristics of the examples studied**

| Example | Number of stories | Story height (m) | Number of spans | Span length (m) | fck (MPa) |
|---|---|---|---|---|---|
| 1 | 16 | 2.90 | 2 | 6.0 | 20 |
| 2 | 20 | Variable | 3 | Variable | 40 |
| 3 | 20 | 2.75 | 4 | Variable | 45 |
| 4 | 30 | 2.85 | 2 | 7.5 | 20 |
| 5 | 16 | 2.88 | 2 | 6.0 | 25 |
| 6 | 15 | 2.90 | 3 | 4.2 | 25 |

metric and physical nonlinearities. The load parcel corresponding to the combination that considered the wind (which acts parallel to the *X* or *Y* axes, depending on the direction of the frame analyzed) was applied as the main variable action. The wind load amounts that the frames received were calculated in function of their lateral stiffness values.

Within the diverse constitutive nonlinear models offered by ANSYS, two stand out as being more appropriate for representing the behavior of concrete: the elastoplastic model based on the Drucker-Prager yield criterion and the specific model for the determination of the failure of brittle materials, obtained using the Willam-Warnke criterion. For the steel, one can choose from between the bilinear or multilinear models, cinematic or isotropic, with or without hardening, according to the Von Mises yield criterion.

The utilization of the model based on the Willam-Warnke failure criterion is limited to a single element, defined as "solid 65". This is a solid three-dimensional element, with eight nodes and three degrees of freedom per node (three translations, in the X, Y and Z directions). It is possible to consider the brittle failure associated with cracking and crushing of the concrete, also admitting consideration of elastoplastic behavior based on Drucker-Prager and Von Mises criteria. There is a possibility of including the reinforcement as a smeared material in the interior of the element, oriented according to three different directions.

In this example the "solid 65" element with smeared reinforcement (in three directions) was utilized in order to represent the columns and beams. The Willam-Warnke criterion allows for the failure condition to be deactivated and replaced with a plastification condition, utilizing, for example, the Drucker-Prager or Von Mises criteria. In the analysis performed, the Willam-Warnke failure criterion was maintained for concrete tension and the Von Mises yield criterion was employed for concrete compression and for the steel. It is important to mention that the Von Mises criterion present, both for concrete and for steel, perfect elastoplastic behavior according to bilinear stress-strain diagrams. In reality, with the goal of avoiding possible numerical difficulties, minimum hardening was considered and a small value was adopted for the tangent modulus in place of zero. It should be noted that the parameters utilized in the nonlinear analyses (materials models, discretization adopted and numeric resources involved) were "calibrated" based on various studies of structural components and reinforced concrete frames that had already been tested experimentally, as was done by Oliveira & Silva [8] and Oliveira [6]. Such studies have revealed the proximity between the experimental results and those obtained from the nonlinear geometric and physical analyses performed using ANSYS, which are therefore considered capable of representing the actual behavior of structures with a good degree of precision.

In order to perform the nonlinear analyses, ANSYS utilizes the Newton-Raphson incremental-iterative method. In this, the number of load increments and the number of iterations for each load step are supplied. Based on a known equilibrium configuration and on load increment data, the structure will respond with a force level below that applied, which results in a residual force that must be applied again, observing the admitted iteration and tolerance limits. In this manner, the stiffness matrix may or may not be updated with each iteration, depending on the option desired by the user. Utilized in these processes were the full Newton-Raphson algorithm, automatic load increments and a limit of 60 iterations per increment, with a tolerance of 0.1% applied to the square root of the sum of the squares of force imbalances.

Nonlinear geometric analyses of the frames were also performed, considering the approximated physical nonlinearity, by means of a reduction in the stiffness of the structural elements. In the case of the columns, the stiffness was reduced by 20% and, for the beams, inertia reduction values were used in accordance with the Branson formula [1] and those recommended in NBR 6118:2007 [2].

Thus, the following values for the effective inertia of the structural elements were adopted:

- $I_{cl} = 0.8\, I_c$ and $I_{bm} = 0.4\, I_c$;
- $I_{cl} = 0.8\, I_c$ and $I_{bm} = 0.5\, I_c$;
- $I_{cl} = 0.8\, I_c$ and $I_{bm} = I_{eq}$ arrived at using equation (2), or that is:

$$I_{eq} = \left(\frac{M_r}{M_a}\right)^3 I_c + \left[1 - \left(\frac{M_r}{M_a}\right)^3\right] I_{II} \leq I_c \qquad (18)$$

In this case, the equivalent inertia values pondered were determined using various applied load percentages (*P*). In this manner, the performance of the Branson inertia equivalent [1] used to consider the physical nonlinearity of the beams in simple form could be assessed for the loads corresponding to the ultimate limit state (defined as 100% *P*), the service limit state (considered to be approximately equal to 45% *P*) and for the unfactored loading (75% *P*).

In order to conduct the nonlinear geometric analyses of the frames using the ANSYS-9.0 software, the columns and beams

**Figure 3 – Load-displacement relationship for the linear elastic analysis performed with the solid model and with the bar models (frame in example 1)**

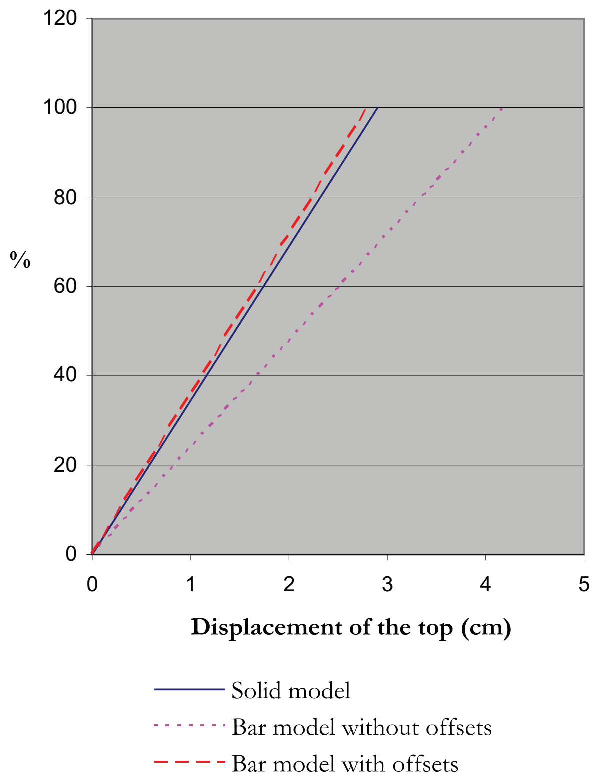



**Figure 4 – Load-displacement relationship for the nonlinear analyses (frame in example 2)**

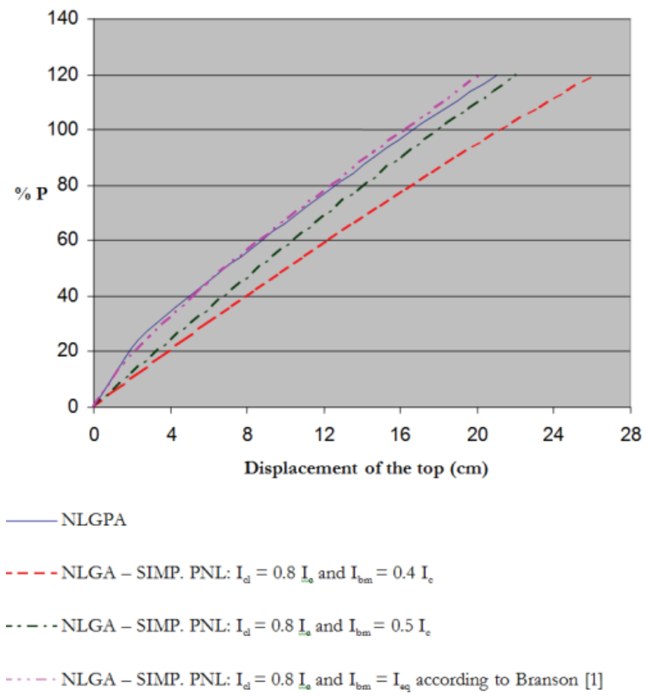
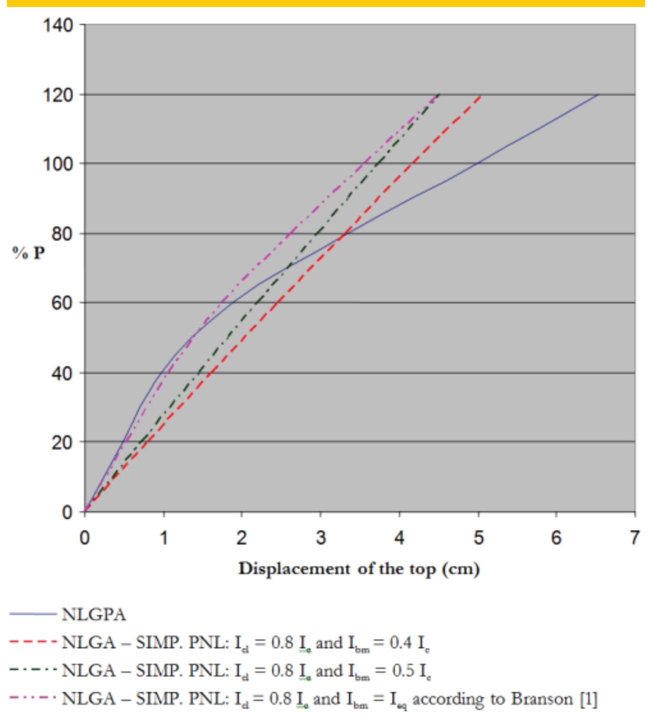


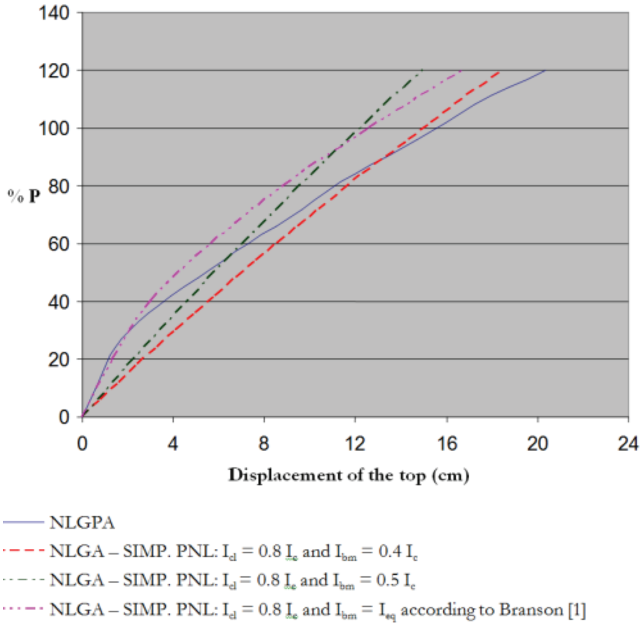

**Figure 5 – Load-displacement relationship for the nonlinear analyses (frame in example 3)**




were represented using the bar elements, with three degrees of freedom in each node: two translations in the X and Y directions and one rotation about the Z axis. It is important to mention that, in the solid model, the beam spans are measured between the column faces, which makes it more rigid than the model for the bars, in which the spans are considered as extending from support axis to support axis. This difference creates a need to utilize the element defined as “beam 54” to represent the extremities of the beams in the bar model. This element allows for the introduction of offsets in the beam-column joint region, making them rigid. As a result, it is possible to compare the two models under equal conditions. It is worth noting that, in order to determine the lengths of the rigid regions to be adopted, comparative analyses (linear elastic) were performed between the solid model and the bar model that utilizes offsets. Figure [3] presents the graph of the applied load *versus* horizontal displacement of the top of the frame in example 1 for the linear elastic analysis performed with the solid model and with the bar models (with and without offsets). It was observed that the bar model without offsets is really much more flexible than the solid model; this, in turn, is much better represented by the bar model that utilizes offsets.

Graphs were created that show the variation in horizontal displacement of the tops of the frames with the load applied, for

Figure 6 – Load-displacement relationship for the nonlinear analyses (frame in example 4)




nonlinear geometric and physical analyses (NLGPA) and nonlinear geometric analyses, with a simplified representation of the physical nonlinearity (NLGA – SIMP. PNL). The graphs for the frames in examples 2, 3 and 4 are presented in figures [4], [5] and [6] (the other graphs, corresponding to examples 1, 5 and 6 can be found in Oliveira [6]). Based on the analysis of the load x displacement graphs, it was possible to visualize the reduction in inertia that better describes the behavior of the frames studied, considering the unfactored loading and the load that correspond to the ultimate limit and service limit states (table [2]).

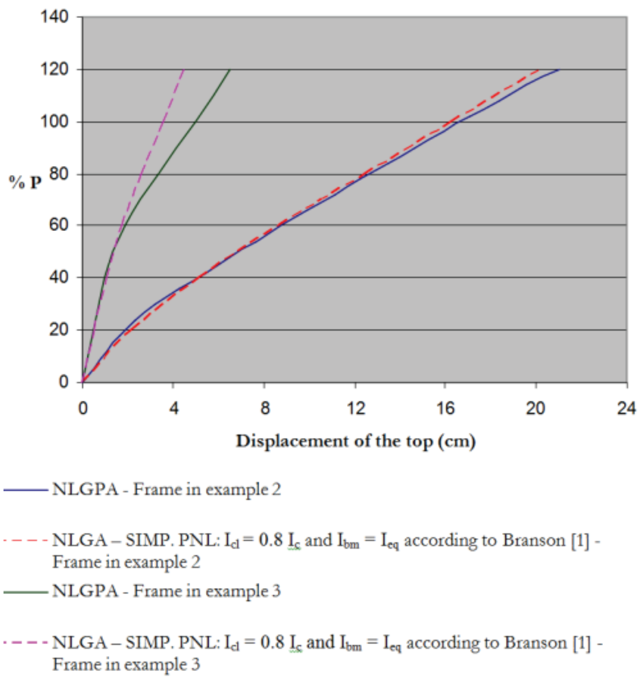
Note that in table [2], in the service limit state, the analyses performed with $I_{cl} = 0.8\ I_c$ and $I_{bm} = I_{eq}$ according to Branson [1] and $I_{cl} = 0.8\ I_c$ and $I_{bm} = 0.5\ I_c$ were shown to be more appropriate for representing the behavior of 83% and 67% of the frames studied, respectively. Therefore, for this load intensity, the analysis that utilizes reductions in inertia equal to $0.8\ I_c$ for the columns and $I_{eq}$ according to Branson [1] for the beams may be considered the most efficient.

Also in table [2], it was observed that, for the unfactored loading and that corresponding to the ultimate limit state, in the majority of the examples analyzed, the utilization of the inertia reduction values adopted in NBR 6118:2007 [2] for the more general cases, or that is, $I_{cl} = 0.8\ I_c$ and $I_{bm} = 0.4\ I_c$, produced the results that best approximated those obtained through the nonlinear geometric and physical analysis. It was also observed that for only examples 2 and 6 the analysis performed using $I_{cl}$ = 0.8 $I_c$ and $I_{bm} = I_{eq}$ according to Branson [1] represented the greatest degree of precision for the "actual" behavior of the structures. It is worth mentioning that the frames in example 2 and 6, among all the frames analyzed, were the most flexible, as can be seen in the graphs in figures [7] and [8]. In these graphs, the load x displacement curves for the frames that present similar heights are confronted. Thus, figure [7] represents the variation in horizontal displacement at the tops of the frames in examples 2 and 3 with the load applied for the nonlinear geometric and physical analyses (NLGPA) and nonlinear geometric analyses that utilize $I_{cl} = 0.8\ I_c$

Table 2 – Inertia reduction that better describes the behavior of the frames studied

| Example | Load corresponding to the service limit state (45% P) | Unfactored loading (75% P) | Load corresponding to the ultimate limit state (100% P) |
|---|---|---|---|
| 1 | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.5 $I_c$ | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.4 $I_c$ | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.4 $I_c$ |
| 2 | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = $I_{eq}$ according to Branson (1) | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = $I_{eq}$ according to Branson (1) | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = $I_{eq}$ according to Branson (1) |
| 3 | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = $I_{eq}$ according to Branson (1) | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.4 $I_c$ | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.4 $I_c$ |
| 4 | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = $I_{eq}$ according to Branson (1)<br>$I_{cl}$ = 0.8 $_{Ic}$ and $I_{bm}$ = 0.5 $I_c$ | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.4 $I_c$ | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.4 $I_c$ |
| 5 | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = $I_{eq}$ according to Branson (1)<br>$I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.5 $I_c$ | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.4 $I_c$ | $I_{cl}$ = 0.8 $I_c$ e $I_{bm}$ = 0.4 $I_c$ |
| 6 | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = $I_{eq}$ according to Branson (1)<br>$I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.5 $I_c$ | $I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.4 $I_c$<br>$I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = 0.5 $I_c$<br>$I_{cl}$ = 0.8 $I_c$ and $I_{bm}$ = $I_{eq}$ according to Branson (1) | $I_{cl}$ = 0.8 $I_c$ e $I_{bm}$ = $I_{eq}$ according to Branson (1) |

**Figure 7 – Load-displacement relationship for the frames in examples 2 and 3**




and $I_{bm} = I_{eq}$ according to Branson [1]. Analogously, figure [8] presents the variation in horizontal displacement of the tops of the frames in examples 5 and 6 with the load applied, also for the nonlinear geometric and physical analyses (NLGPA) and nonlinear geometric analyses that utilize $I_{cl} = 0.8\ I_c$ and $I_{bm} = I_{eq}$ according to Branson [1].

In an analysis of the graphs in figures [7] and [8], it can be seen that, in fact, the utilization of inertia values equal to $I_{cl} = 0.8\ I_c$ and $I_{bm} = I_{eq}$ according to Branson [1] is much more efficient for the representation of the behavior of the more flexible frames in examples 2 and 6 than for the stiffer frames in examples 3 and 5. This is certainly due to the Branson inertia equivalent formula [1] itself, which consists of a ponderation of the inertias in the gross concrete section (state I) and cracked concrete section (state II). The greater the $M_a/M_r$ relationship, the greater is the contribution from inertia in the cracked section $I_{II}$. In the event the moment $M_a$ is smaller than the cracking moment $M_r$, the inertia from the gross section of concrete $I_c$ is adopted for Branson's inertia equivalent [1].

The frames in examples 2 and 6 present beams with lower inertia and have cracking moments $M_r$ that are greatly inferior to those of the frames in examples 3 and 5, as is shown in table [3]. This means that the moments $M_a$ will surpass the cracking moments $M_r$ much more quickly for the frames in examples 2 and 6 than for the frames in examples 3 and 5; in the latter, therefore, Branson's inertia equivalent [1] approximates the inertia for the gross section of concrete, even for greater load intensities, which may result in values that do not translate the actual loss of stiffness in the structure. In the case of the frames in examples 2 and 6, based on the lower load values, the equivalent inertia shall be determined, in large part, by the inertia for the cracked section $I_{II}$, which is coherent for structures with less stiffness and, therefore, with a greater cracking intensity.

It should be mentioned that, considering the small load intensities, from which the structures had not yet cracked, the analyses performed using $I_{cl} = 0.8\ I_c$ and $I_{bm} = I_{eq}$ according to Branson [1] represent a good degree of precision regarding the behavior of all the frames, both the more flexible ones and the stiffer ones (figures [7] and [8]). This is predictable, since, for the small load values $P$, the bending moments $M_a$ are inferior to the cracking moment $M_r$ and, consequently, the inertia value for the gross section of concrete $I_c$ is adopted for Branson's inertia equivalent [1].

## 5. Final considerations

This paper attempts to evaluate the efficiency of Branson's inertia equivalent [1] to consider the physical nonlinearity of beams in simple form. To this end, various numerical analyses of plane frames belonging to regular reinforced concrete buildings were performed utilizing ANSYS software. Initially, the frames were processed considering both the geometric and physical nonlinearities. Next, nonlinear geometric analyses were performed considering an approximated physical nonlinearity, by means of a reduction in the stiffness of the structural elements. In the case of the columns, the stiffness was reduced by 20% and, for the beams, the inertia reduction values according to the Branson formula [1] and those recommended in NBR 6118:2007 [2] were utilized. The results of the nonlinear geometric analyses, considering the simplified physical nonlinearity, were then compared

**Figure 8 – Load-displacement relationship for the frames in examples 5 and 6**

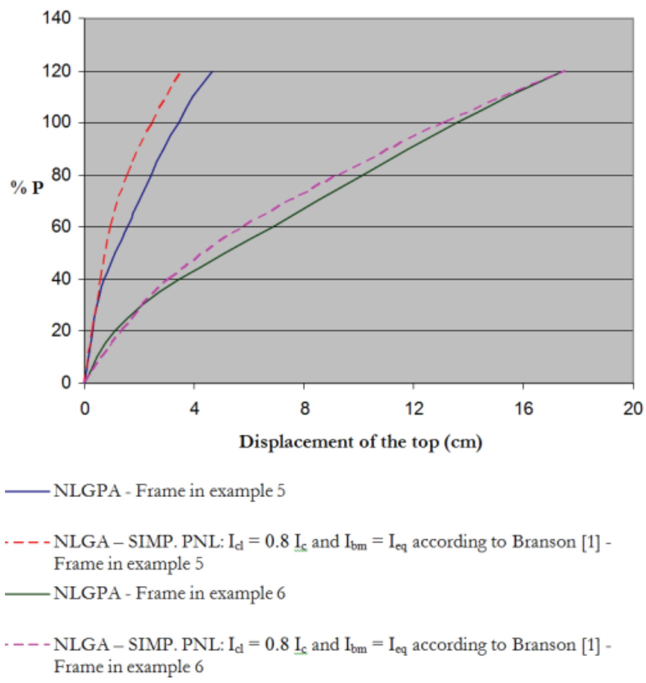


Table 3 – Cracking moments of beams of the frames in examples 2, 3, 5 and 6

| Example | Cracking moment $M_r$ (kN · cm) |
|---|---|
| 2 | 2625 |
| 3 | 6840 |
| 5 | 6370 |
| 6 | 2080 |

with those obtained using the nonlinear geometric and physical analyses, which are capable of representing the actual behavior of structures with greater precision.

The performance of the nonlinear geometric analyses, considering the simplified physical nonlinearity, was evaluated for the loads corresponding to the ultimate limit state (defined as 100% *P*), the service limit state (considered approximately equal to 45% *P*) and for the unfactored loading (75% *P*).

In the service limit state, the analysis that utilizes inertia reductions equal to $0.8\ I_c$ for the columns and $I_{eq}$ according to Branson [1] for the beams may be considered the most efficient.

For the unfactored loading and the one corresponding to the ultimate limit state, the majority of the examples analyzed, the utilization of the inertia reduction values adopted in NBR 6118:2007 [2] for the more general cases, or that is, $I_{cl} = 0.8\ I_c$ and $I_{bm} = 0.4\ I_c$, produced the closest results to those obtained through the nonlinear geometric and physical analysis.

It is worth mentioning that the utilization of values equal to $I_{cl} = 0.8\ I_c$ and $I_{bm} = I_{eq}$ according to Branson [1] was shown to be more efficient for representing the behavior of the more flexible frames than for the stiffer frames. This is certainly due to the Branson inertia equivalent formula [1] itself, which for the stiffer frames, approximates the inertia of the gross concrete section, even for greater load intensities, and may result in values that do not translate the actual loss of stiffness in the structure. In the case of the more flexible frames, based on the lower load values, the inertia equivalent shall be determined, in large part, by the inertia of the cracked section $I_{II}$, which is coherent for the structures with smaller stiffness values and, thus, with greater cracking intensity.

It was also noted that, for small load intensities, under which the structures had not yet cracked, the analyses performed using $I_{cl} = 0.8\ I_c$ and $I_{bm} = I_{eq}$ according to Branson [1] represented the behavior of all the frames, both the more flexible ones and the stiffer ones, with a good degree of precision. This can be explained by remembering that, for small load values, the bending moments $M_a$ are lower than the cracking moment $M_r$ and, consequently, the inertia values for the gross section of concrete $I_c$ is adopted for the Branson inertia equivalent [1].

Finally, starting with the principle that the stiffness reduction coefficients for the structural elements are normally aimed at normal building projects, generally dimensioned for the load corresponding to the ultimate limit state, one may consider the inertia reductions as being equal to $0.8\ I_c$ for the columns and $0.4\ I_c$ for the beams as being the most representative of the behavior of the frames analyzed. It should also be noted that the utilization of a constant coefficient for all the beams results in a simpler procedure that is practical and easy to employ and is extremely advantageous compared with the utilization of the Branson inertia equivalent [1], which represents different values for each span and for each story of the building structure.

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