

# Numerical and experimental analysis of the displacements evolution of reinforced concrete beams under repeated cyclic loads

## *Análise numérica e experimental da evolução de flechas de vigas de concreto armado sob ações cíclicas repetidas*



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### Abstract

This paper discusses the structural behavior of reinforced concrete beams subjected to cyclic loadings by numerical and experimental analysis. The main objective was to quantify the increase in deflections of the beams when subjected to repeated loading cycles. Experimental tests with the application of cyclic and monotonic loading were carried out with reduced size beams. The numerical analysis utilized a simplified damage model that is part of the group denominated lumped dissipation models. This model takes into account the damage increase as a function of the increase in the number of cycles, enabling an evaluation of the stiffness loss due to repeated loads. The experimental results not only confirmed the effect of repeated loads on the stiffness loss of beams, but also demonstrated the important influence of the flexural reinforcement ratio on cyclic behavior. Comparisons between the experimental results and those obtained with the numerical model provided support for the potential of the damage model employed in the prediction of the increase of deflections caused by repeated cyclic loading.

**Keywords:** beams, reinforced concrete, cyclic loads, damage; stiffness loss.

### Resumo

O presente trabalho discute o comportamento estrutural de vigas de concreto armado submetidas a carregamentos cíclicos, através de análise numérica e experimental. O objetivo principal é quantificar o crescimento das flechas dessas vigas quando submetidas a ciclos de carga repetida. Foram realizados ensaios experimentais com vigas de tamanho reduzido, com aplicação de carregamento monotônico e cíclico. A análise numérica utiliza um modelo simplificado de dano que se enquadra nos denominados modelos de dissipação concentrada. O referido modelo leva em conta o acréscimo de dano em função do aumento do número de ciclos, podendo assim avaliar a perda de rigidez das vigas decorrente das ações repetidas.

Os resultados experimentais confirmaram não somente o efeito das ações repetidas na perda de rigidez das vigas, mas também reforçam a importante influência da taxa de armadura de flexão no comportamento cíclico. Comparações entre os resultados experimentais e os obtidos com o modelo numérico forneceram indícios do potencial do modelo de dano empregado para a previsão do crescimento de flechas com o carregamento cíclico repetido.

**Palavras-chave:** vigas, concreto armado, ações cíclicas, dano, perda de rigidez.

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## 1. Introduction

Despite the significant development achieved in materials science and structural engineering during the last decades, the behavior of concrete is still considered as highly complex. The nonlinear behavior of concrete resulting from cracking starts even before its use with the micro cracking of the mortar matrix during curing process. This behavior is very difficult to be represented by numerical models even for monotonically loaded elements, and mainly for cyclic loading.

Reinforced concrete behavior is highly affected by the steel-concrete bond behavior, which is dependent of the type of applied load. Cyclic loading are characterized by the variation of stresses within a certain range with time. They provoke bond decay and strain increase due to the evolution of cracking process.

Many studies have been conducted in the last years to investigate the influence of cyclic loads on reinforced concrete structures (including those using composite materials as reinforcement) mainly for the ultimate limit state of fatigue [1-4] and for the degradation of the steel-concrete bond [5-9]. In Brazil, Braguim [10] and Oliveira Filho [11] studied the behavior of reinforced concrete beams under repeated loading.

However, even with those numerous results, most of countries, including Brazil, have codes whose procedures are based on researches considering only monotonic loading.

So, to understand the behavior of each component material, steel and concrete, and the behavior of both, forming a composite structure, still remain as challenges to be faced and motivation for new studies.

Present work has two main objectives: i) to present experimental results showing the increase of deflections due to cyclic loading on reinforced concrete beams, and ii) to propose a simple and consistent procedure to predict the deflection evolution during cycles, considering the influence of important parameters as reinforcement ratio.

The experimental results presented in this paper were obtained for the PhD thesis of Oliveira Filho [11] who performed an experimental investigation of the behavior of reinforced concrete beams submitted to monotonic and cyclic loads.

The theoretical model proposed in this paper used a damage model based on Picón and Flórez-López [12]. A comparison between theoretical and experimental results allowed the evaluation of the potential of the proposed model for simulations with repeated loads.

## 2. Tests on reinforced concrete beams

Tests on reinforced concrete beams under repeated cyclic loading were performed. In these tests, it was intended to determine the loss of rigidity caused by the imposed cyclic loading measuring vertical displacements and strains on concrete surface (compression zone) and in reinforcing bars. The used equipment consisted of an INSTRON testing machine with a servo-hydraulic actuator of 500kN capacity, from the Structural Laboratory of Sao Carlos Engineering School.

In this paper, they are presented the tests results on seven beams subjected to cyclic loading and subsequent unloading and monotonic loading till failure.

Figure 1 - Geometry and reinforcement layout of tested beams with rectangular cross section (dimensions in mm)

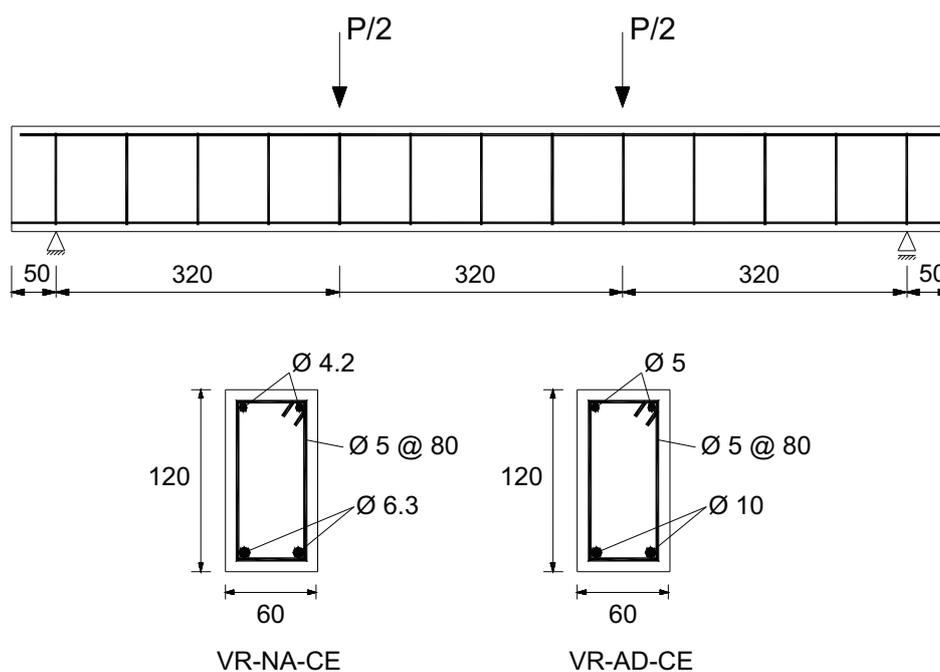
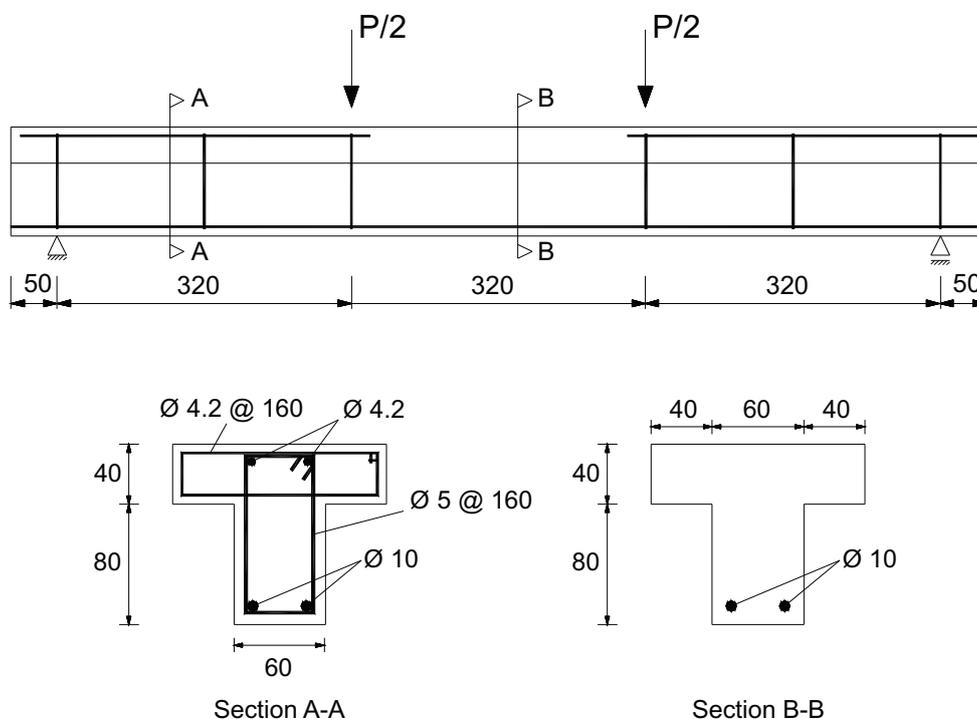


Figure 2 - Geometry and reinforcement layout of tested beam VT-NA-SE-02 (dimensions in mm)



The beams were made in a reduced scale, with rectangular and T crosses sections. Figures 1 and 2 show the geometry, the reinforcement details and the loading scheme used for tests.

The beams were divided into 3 groups (I, II e III):

Group I consisted of 3 beams with rectangular cross section, designed as normally reinforced ones (VR-NA), in the limit of do-

mains 2 and 3, with stirrups (CE). First tested beam of the group was used as reference to evaluate the failure load and also to determine the frequency to be used in the cyclic loading of the 2 remaining beams (VR-NA-CE-01 and VR-NA-CE-02).

Group II consisted also of 3 beams with rectangular cross section, designed with additional longitudinal compression reinforcement

Figure 3 - Strain gauges location

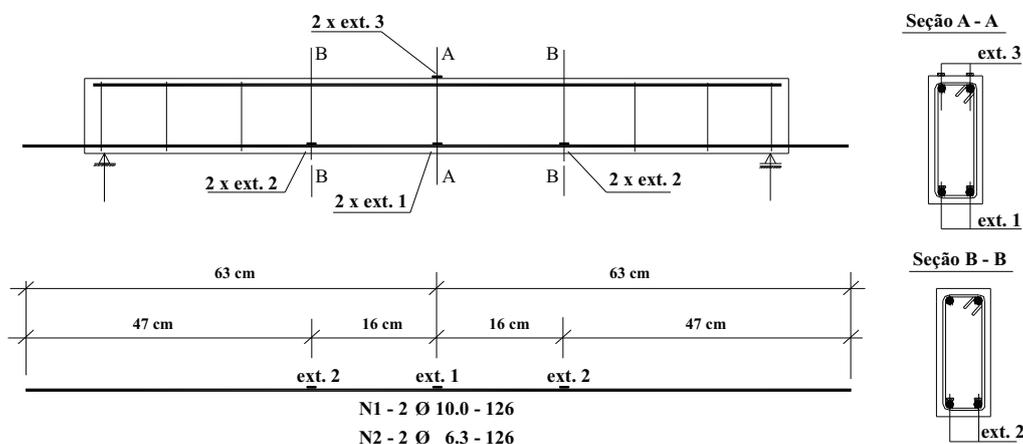
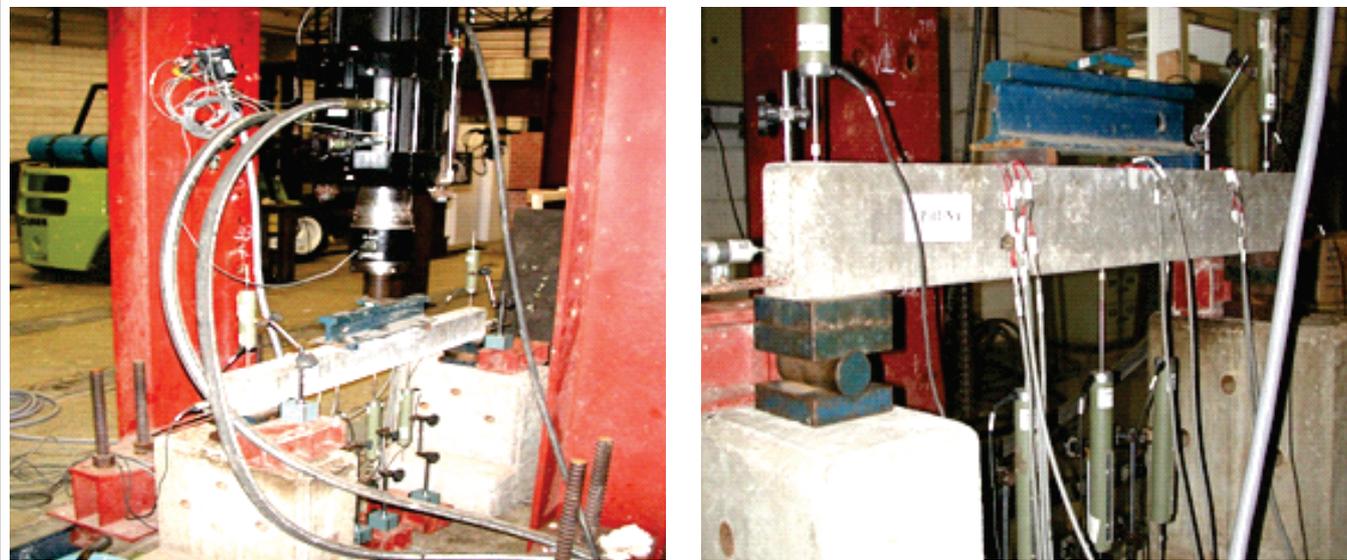


Figure 4 - Test frame, application of loads and instrumentation details



(VR-AD), to avoid neutral axis in Domain 4, and with stirrups (CE). As for the Group I, first tested beam of Group II (VR-AD-T) was taken as reference to evaluate the load capacity. Finally, Group III consisted of only one beam of T cross section, designed as normally reinforced (VT-NA), without stirrups (SE) in the zone of constant bending moment. Steel wires of 4.2 mm and 5.0

Table 1 - Failure of reference beams

Beams	Failure Load (kN) - Predicted	Failure Load (kN) - Test	Failure Mode - Test
VR-NA-T	21.0	22.0	(1)
VR-AD-T	52.0	55.4	(1)

(1) Concrete crushing failure with yielding of flexural reinforcement

mm were made of CA-60 and bars of 6.3 mm and 10.0 mm were CA-50.

Regarding concrete properties, tests performed gave the following mean values: compressive strength of 40.19 MPa, secant elasticity modulus of 26781 MPa and tension strength (from Brazilian test) of 3.20 MPa.

Measurements system consisted of electrical strain gauges placed on reinforcement

Table 2 - Test parameters of beams (groups I, II and III): cyclic and failure stages

Beam	$P_{max}$ (kN)	$P_{min}$ (kN)	Number of cycles	$P_{u,theor}$ (kN)	$P_{u,exp}$ (kN)	Failure Mode - Test
VR-NA-CE-01	18	8	25000	21	22.0	(1)
VR-NA-CE-02	20	8	15000	21	25.0	(1)
VR-AD-CE-01	45	20	30000	52	56.2	(1)
VR-AD-CE-02	47	20	30000	52	56.6	(1)
VT-NA-SE-02	44	20	30000	52	57.6	(1)

(1) Concrete crushing failure with yielding of flexural reinforcement

$P_{u,theor}$  = Predicted failure load

$P_{u,exp}$  = Experimental failure load

**Figure 5 – Failure mode for beams of group II (VR-AD-CE)**



and concrete, for both groups of beams. Strain-gauges 1 and 2 were placed on reinforcement and 3 on the upper face of concrete beams, as shown in Figure 3.

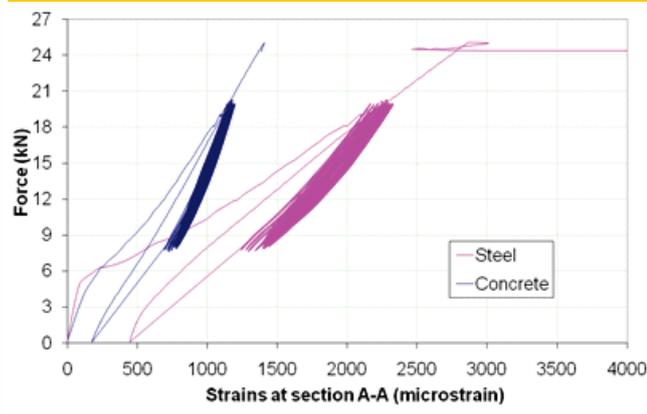
Figure 4 shows the test frame with the beam, the hydraulic actuator, the load cell and also the used instrumentation, supports and load system.

At first, reference tests (T) were performed to determine the failure load. One beam of each group, I and II, were tested. The predicted failure loads and the obtained values are given in Table 1.

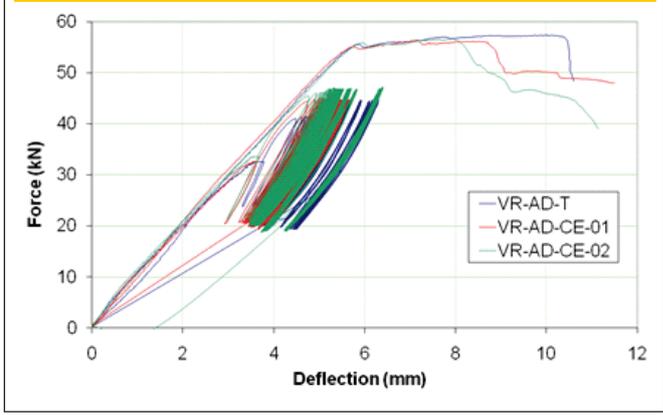
Table 2 shows a summary of main tests parameters for the remaining beams of groups I, II and III.

Predicted failure loads were obtained from usual calculation for reinforced concrete cross sections under flexure at ultimate limit state. Mean values of materials strength and deformability obtained in the characterization tests of steel and concrete were used in this calculation.

**Figure 6 – Strains of concrete and flexural tensile reinforcement: beam VR-NA-CE-02**



**Figure 7 – Evolution of the deflections of the beams: Group II**



The minimum value (lower limit) defined for cyclic loading (38% of predicted failure load) intended to represent the long-term loads of structures subjected to repeated loads, as in bridges or viaducts. The maximum value (upper limit) intended to represent a situation that occurs many times during the life time of a structure subjected to repeated loads (around  $10^5$  times). In bridges, for example, this situation corresponds to long-term loads plus live loads, being represented by the Frequent Load Combination considered in the Service Limit States.

The total number of cycles used in the tests was defined by the operational limit- the maximum number of cycles possible in one day of work, without interrupting the actuator operation and the tests readings. In this way, the tests represented around 15% to 30% of repetitions that characterize the Frequent Load Combination.

The frequency of loading was from 1.0 to 2.0 Hz. The acquisition data rate permitted approximately twenty readings per cycle.

After cyclic loading, the beams were taken till failure, being observed in all of them the rupture of compressed concrete and the yielding of tensile reinforcement. Figure 5 shows the failure mode of beams from group II (VR-AD-CE).

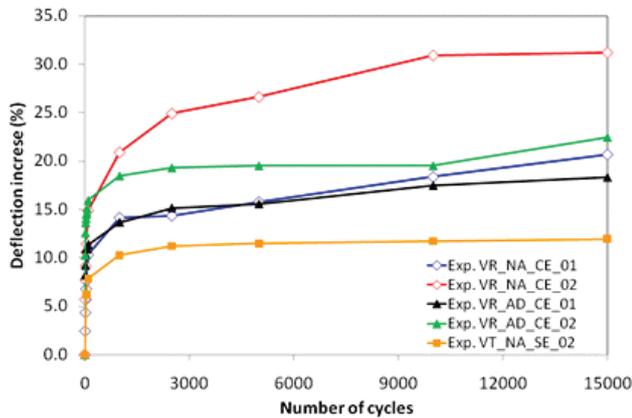
It was possible to notice the increase of materials strain and also the increase of beams deflections with the loading cycles, as indicated in Figures 6 and 7. Figure 6 shows the evolution of strain during cyclic loading stage and just before failure, for beam VR-NA-CE-02. Figure 7 shows the evolution of the deflections of the beams from group II at cyclic stage and after that, when they were taken to rupture.

Graphic of figure 8 shows the percentage of maximum deflection increase for all five beams, analyzed at cyclic stage after 15000 cycles. It can be noticed that beams with lower reinforcement rate- the ones of group I (VR-NA-CE) – presented higher deflection increase than others, reassuring the influence of the amount of reinforcement in the amplification of deflections with cyclic loading.

### 3. Considered theoretical model

The used theoretical model was based on Cipollina and Flórez-López [13] and Picón and Flórez-López [12], and is applicable to reinforced concrete linear structures under flexure. The referred models are based on Damage Mechanics and are the so-called Lumped

**Figure 8 – Increase of maximum deflection related to the first cycle**



Dissipation Models. These models consider as a simplification that all dissipative processes of damage of concrete and plasticity of reinforcement occur in *hinges* of null length at the endings of the element, with the behavior remaining linear elastic in the rest of it. So, the material nonlinearity is represented by damage variables at the endings of elements ( $d_i$  and  $d_j$ ) – which take into account not only the loss of flexural rigidity produced by the cracking of concrete (Figure 9) but also the plastic rotations at the endings of element, consequent of plastic deformations at tensile reinforcement.

A moment-rotation curve of a reinforced concrete cross section given in Figure 10 shows the main entry parameters of this type of model: cracking moment ( $M_c$ ), plastic moment ( $M_p$ ), ultimate moment or resistant moment ( $M_u$ ) and the ultimate plastic rotation ( $\theta_{pu}$ ) associated to the plastic rotation capacity. Such parameters can be obtained from the usual calculation for reinforced concrete cross sections.

In this work, at first is presented the formulation for the case of monotonic loads and cyclic loads without fatigue, which was the basis for the formulation applicable to repeated cyclic loading considering the loss of rigidity due to the cycles of loading.

Regarding monotonic or cyclic loading without fatigue, the model proposed by Alva [14, 15] was used, producing an improvement on the models proposed before by Cipollina and Flórez-López [13] e Flórez-López [16]. They proposed two functions to consider non-linear effects: a limit function to control damage evolution, and another one to control the evolution of plastic rotations. The second function will not be considered in this work since it cannot take into account the permanent deformation produced by cyclic loading at a level lower than the plastic moment.

Regarding this limitation and also considering the amplitude of the cyclic loads applied to the beams in the experimental program (lower than the needed to cause the yielding of tensile reinforcement), a non-linear elastic model will be adopted for the numerical analysis, being considered only the functions that control the evolution of damage.

### 3.1 Formulation for monotonic or cyclic loads without fatigue

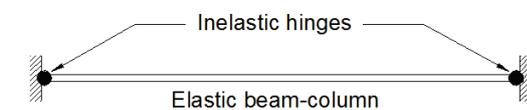
The limit function that controls the evolution of damage variable, for each element end, is given by:

$$g = G - R \tag{1}$$

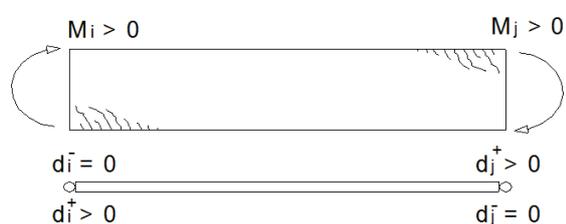
With:

$$G = \frac{1}{2S} \left( \frac{M}{1-d} \right)^2$$

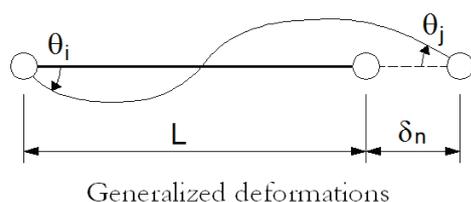
**Figure 9 – Lumped Dissipation Model**



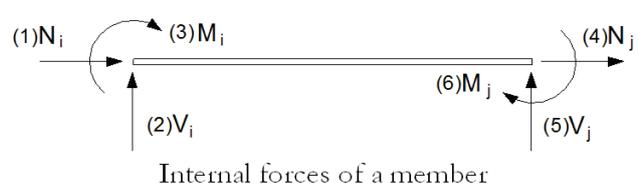
Lumped dissipation model of a member



Representation of state of damage in RC member

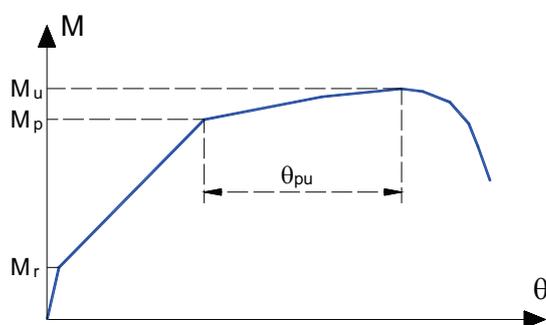


Generalized deformations



Internal forces of a member

Figure 10 - Main parameters of Lumped Dissipation Models



**Model parameters**

- $M_r$  = cracking moment
- $M_p$  = plastic moment
- $M_u$  = ultimate moment
- $\theta_{pu}$  = plastic rotation at ultimate moment (plastic rotation capacity)

$$R = G_{cr} - e^{-\gamma(1-d)} \cdot q \frac{\ln(1-d)}{(1-d)}$$

$$S = 4EI/L$$

$$G_{cr} = \frac{M_r^2}{2S}$$

where:

G is the thermodynamic moment and R represents the cracking resistance term;

M is the bending moment at the element end;

d is the value of damage at the element end;

EI is the flexural stiffness of the undamaged element (without cracking);

L is the length of the element;

$M_r$  is the cracking moment;

$G_{cr}$  is the value of G when  $M=M_r$  and  $d=0$ ;

$\gamma$  is a parameter (non-dimensional) that controls the loss of flexural rigidity after the beginning of the cracking process. It is an entry parameter for the model and it depends specially of the tensile reinforcement ratio (that is,  $\gamma$  it is not a free parameter). The lower the reinforcement rate, the higher the loss of rigidity and thereafter, the higher is the value of the parameter  $\gamma$ .

Constant q is obtained from the condition  $g=0$  and also considering that the moment is maximum when  $M=M_u$  (ultimate moment or resistant moment). The detailed calculation of this parameter is found in ALVA [14, 15, 17]. The evolution of damage variable occurs according to the following conditions:

$$\begin{aligned} \Delta d &= 0 && \text{if } g < 0 \text{ or } dg < 0 \\ \Delta d &\neq 0 && \text{if } g = 0 \text{ or } dg = 0 \end{aligned} \quad (2)$$

Based on such conditions, it can be obtained the increment of the damage variable:

$$\Delta d = \frac{\langle dg \rangle}{\frac{\partial R}{\partial d}} \quad (3)$$

where

$$\frac{\partial R}{\partial d} = \gamma \cdot e^{-\gamma(1-d)} \cdot q \cdot \left[ \frac{h(1-d)}{(1-d)} \right] + e^{-\gamma(1-d)} \cdot q \cdot \left[ \frac{h(1-d)-1}{(1-d)^2} \right]$$

The stiffness matrix for a plane frame element with 6 degrees of freedom, including the damage effects at the endings ( $d_i$  e  $d_j$ ) is presented in ÁLVARES [18] and in ALVA [14,15].

**3.2 Formulation proposed for repeated cyclic loads with fatigue**

For cyclic loads in general, the additional damage of materials occurs not only as a function of the level of loading, but also is function of the cycles. For reinforced concrete beams, the deleterious effects of cyclic loading can be related to the rupture by fatigue of concrete, fatigue of reinforcement or also to the loss of steel-concrete bond.

In this case, it can be mentioned the model proposed by Picón and Flórez-López [12], applicable to the case of reversed cyclic loading. The referred model uses a formulation similar to the one presented by Cipollina and Flórez-López [13] and Flórez-López [16], but including an additional parameter in the law of the damage evolution. In this way, the increment of the damage variable is obtained by:

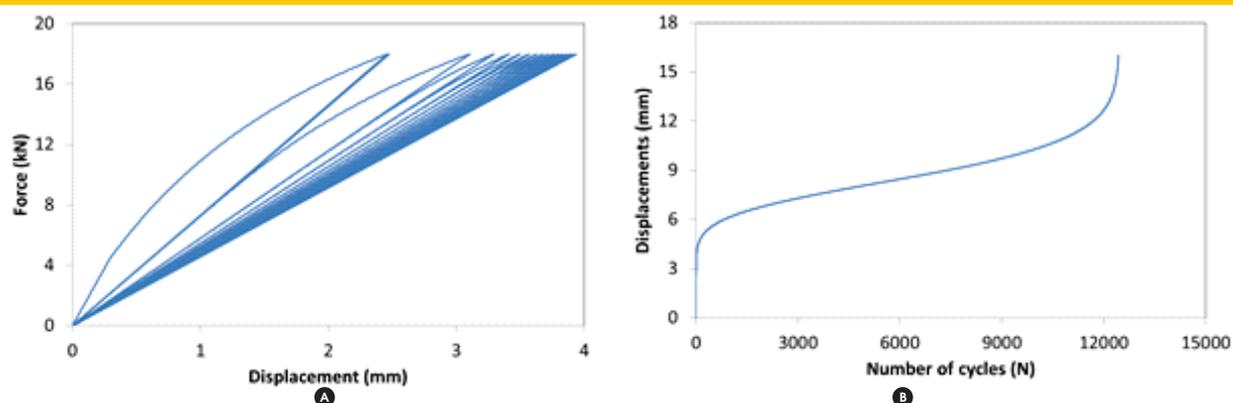
$$\Delta d = \frac{G^z}{R^z} \frac{\partial R}{\partial d} \langle dG \rangle \text{ if } G \geq G_{cr} ; \Delta d = 0 \text{ if } G < G_{cr} \quad (4)$$

where z is a parameter that controls the increment of damage caused by fatigue.

Picón and Flórez-López [12] propose parameter z as a variable of the numerical analysis, depending on the produced level of damage, according a second degree function. It is worth to point out that *the lower the values of z are, higher are the effects of loss of rigidity due fatigue* (that is, provoked by the cycles of loading).

Based on the experimental results, including the ones given by Oliveira Filho [11]- and also in numerous numerical analysis, it is

Figure 11 – Results obtained by proposed numerical model for reinforced concrete beam subjected to vertical cyclic loads: a) Load vs. Deflection; b) Deflection evolution with increasing number of cycles



proposed in this work that the parameter  $z$  is determined from the following general expression for the case of repeated cyclic loading:

$$z = A.d^4 \quad (5)$$

where  $A$  is a constant that must be obtained from cyclic loading tests. Figure 11 shows an example of the numerical answer given by the nonlinear elastic model for reinforced concrete beam subjected concentrated loads in the thirds of span. This specific beam has the same geometric and mechanical properties of VR-NA-CE-01, considered in item 2, however this one was submitted to repeated cyclic load with an amplitude corresponding to 80% of the predicted failure load, being the minimum load equal to zero.

The results presented in Figure 11 indicate that the numerical model considers the loss of rigidity due to the cyclic loading. Figure 11b shows the deflection- number of cycles curve characterized by 3 different stages. In the first, the increase of deflection is considerable, and it tends to stabilize in the second stage. Finally, the third one shows the proximity of the beam failure, where the deflections increase significantly- coherent with can be observed in the tests of materials submitted to the process of low-cycle fatigue or high-cycle fatigue.

Even though the numerical model has potential to represent a fatigue failure (Figure 11), the term *fatigue* used to refer to the proposed model is only associated to the damage (loss of flexural rigidity) caused by the loss of bond between steel and concrete during repeated cyclic loading.

The present theoretical model with the proposed expression for  $z$  (Equation 5) was implemented in a computer program for nonlinear analysis of plane frames with force control using FORTRAN language. To solve the nonlinear problem, it was used the incremental and iterative procedure of Newton-Raphson (standard), where the tangent stiffness matrix is updated at each iteration. The used convergence criteria considered the norm of residual unbalanced forces.

#### 4. Comparisons and analysis of results

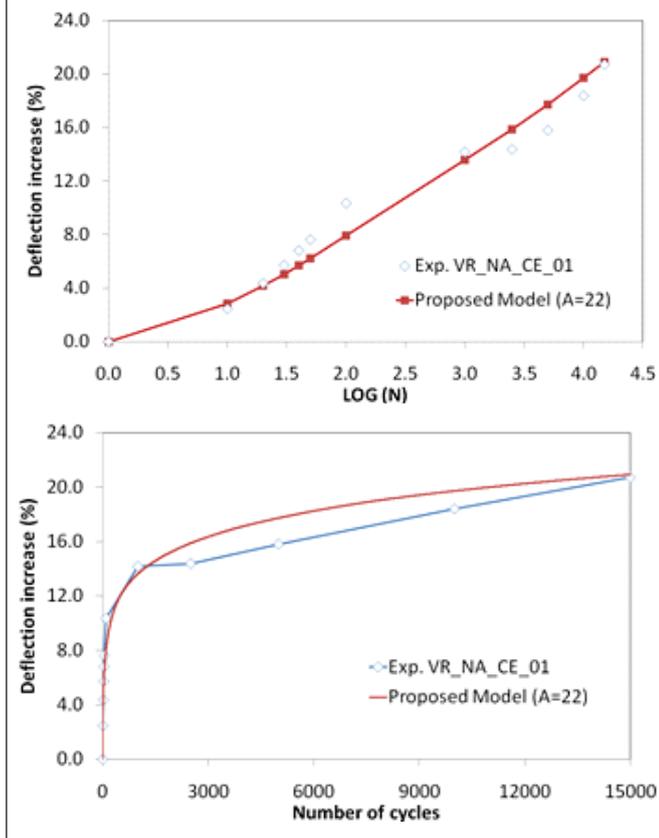
In this work, the experimental results of 5 beams tested by Oliveira Filho [11] were compared to the theoretical results given by the damage model. Table 3 contains the entry parameters of numerical model and the number of cycles used for comparison with experimental results.

For beam VR-AD-CE-02 comparisons between experimental and theoretical results were limited to the cycle 5000, when a horizontal displacement of few centimeters was observed in one of the supports. For beam VR-NA-CE-01, the effects of the support displacements could be corrected from the collected experimen-

Table 3 – Numerical model parameters and number of cycles analyzed

Beam	$M_r$ (kN.cm)	$M_p$ (kN.cm)	$M_u$ (kN.cm)	$\gamma$	Number of cycles
VR_NA_CE_01	71	317	376	3.5	15000
VR_NA_CE_02	71	317	376	3.5	15000
VR_AD_CE_01	71	880	902	1.0	15000
VR_AD_CE_02	71	880	902	1.0	5000
VT_NA_SE_02	98	888	922	2.5	15000

**Figure 12 – Theoretical and experimental results – Beam VR-NA-CE-01**



tal results and also from the analysis of observed tendency of deflections increase.

Parameters  $M_r$ ,  $M_p$  and  $M_u$  were obtained from the experimental results. For a good prediction of parameter  $\gamma$ , numerical simulations using a more precise model were performed. Such model consisted of the discretization of the beam in 12 elements using moment-curvature diagrams to obtain the flexural stiffness of elements. For behavior in service, Branson expression was used. Behavior at ultimate limit state was represented by a linear branch for moment-curvature diagram between the beginning of reinforcement plastic behavior and the ultimate moment. Considering the load-deflection diagrams given by the damage model and also by the refined model, values for  $\gamma$  were obtained and are shown in Table 3.

Graphics of Figures 12 to 16 indicate the increase of beams deflections along the cycles of loading (related to the first cycle). In those figures are shown the most adequate values of constant A regarding the experimental results.

The graphics of Figures 12 to 16 show that the proposed model presents a good potential to predict the stiffness loss and the deflection increase that occurs in reinforced concrete beams subjected to repeated cyclic loading. However, it is necessary a careful evaluation of the values assumed by constant A in function of the many variables affecting the problem solution.

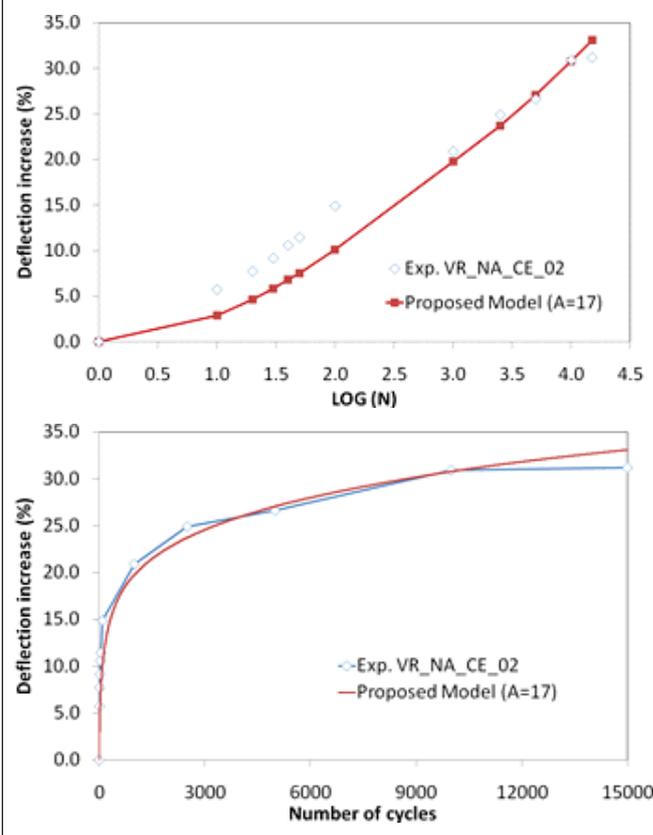
Table 4 contains the values of the damage variables along the load cycles and the values of A that best fitted the experimental results. The main variable parameter for the beams of group VR-AD and group VR-NA was the geometric reinforcement rate. Comparison between graphics of Figures 12 and 13 with the ones of Figures 14 and 15 showed that, in general, the increase of tensile reinforcement reduced the stiffness loss caused by cyclic loading. Regarding the numerical model, increase of the reinforcement produced higher values of constant A that best fitted the experimental results. Comparing results of beam VT-NA-SE-02 with beams of group VR-NA, similar conclusions can be drawn regarding the influence of reinforcement rate.

Beams of group VR-AD had a reinforcement rate about two and half times the beam of group VR-NA. It is worth to notice that the values of A for beams of group VR-AD are higher than the ones for beams of group VR-NA approximately at the same proportion (about 2.3 times).

However, the relation between the amplitude of loading and the failure load was not the same for beams of groups VR-NA and VR-AD, varying from 0.445 to 0.480 if the experimental failure load is considered, and from 0.476 to 0.571 if the theoretical value is considered. For beam VT-NA-SE-02, this relation was 0.347 for experimental load and 0.385 for theoretical value.

It must be reassured that for general materials the higher the am-

**Figure 13 – Theoretical and experimental results – Beam VR-NA-CE-02**



plitude of loading the higher is the damage along the cycles. In this way, for the same reinforcement rates, it is expected that the higher is the amplitude the lower is the value of  $z$  for the numerical model (and therefore, lower the value of constant  $A$ ).

To improve the analysis regarding the involved variables and also to obtain an estimative of the magnitude of constant  $A$  for more general cases, two assumptions will be made:

- a) Value of constant  $A$  is directly proportional to the reinforcement rate;
  - b) Value of constant  $A$  is inversely proportional to the load amplitude.
- Within the adopted assumptions, the value for  $A$  can be given by:

$$A = k \cdot \rho \cdot \frac{P_u}{\Delta P} \quad (6)$$

where:

$r$  is the geometric tensile reinforcement rate referred to concrete gross cross section;

$P_u$  is the failure load (experimental or theoretical) of monotonic loading;

$\Delta P$  is the load amplitude;

$k$  is a proportionality constant, obtained from the best fitted  $A$  values regarding experimental results.

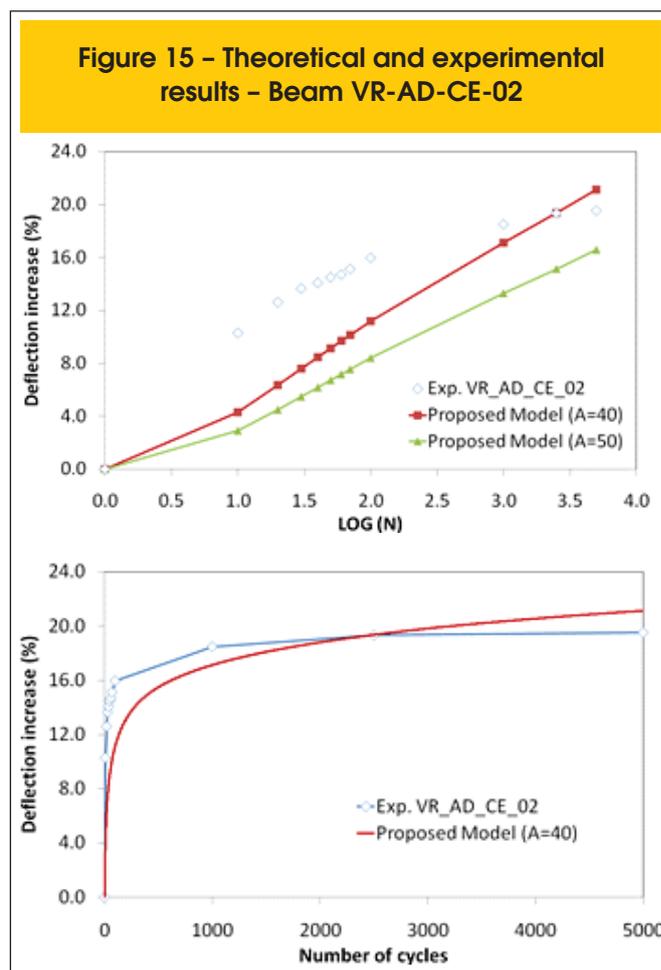
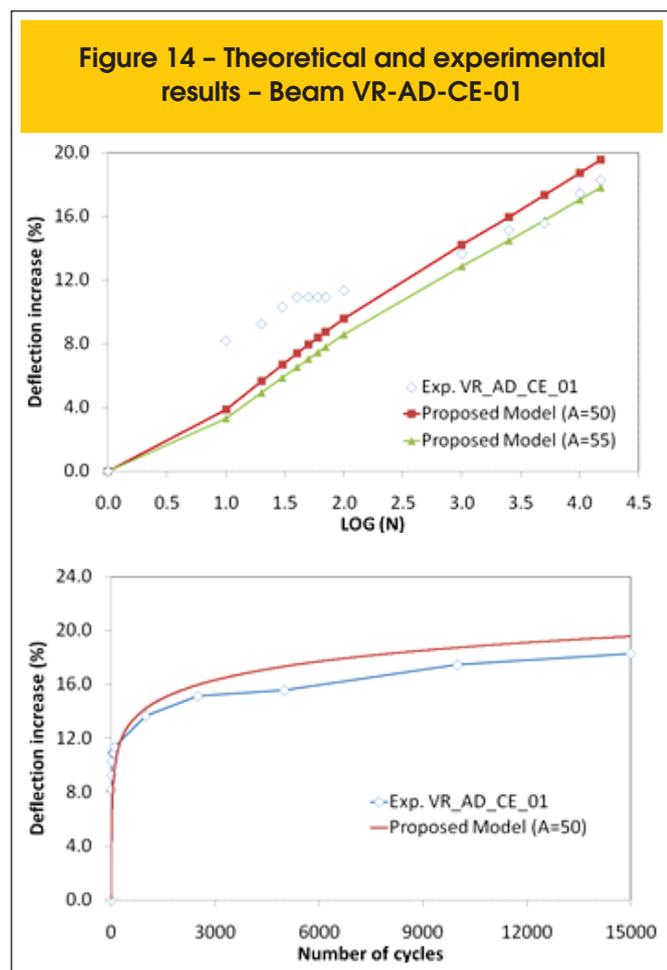
Table 5 contains the experimental values of constant  $k$  obtained from Equation (6).

The results given by Table 5 indicate that values of  $k$  between 1000 and 1200 represent an initial estimative for parameter  $A$  of the theoretical model, for beams of similar characteristics of the considered in this work..

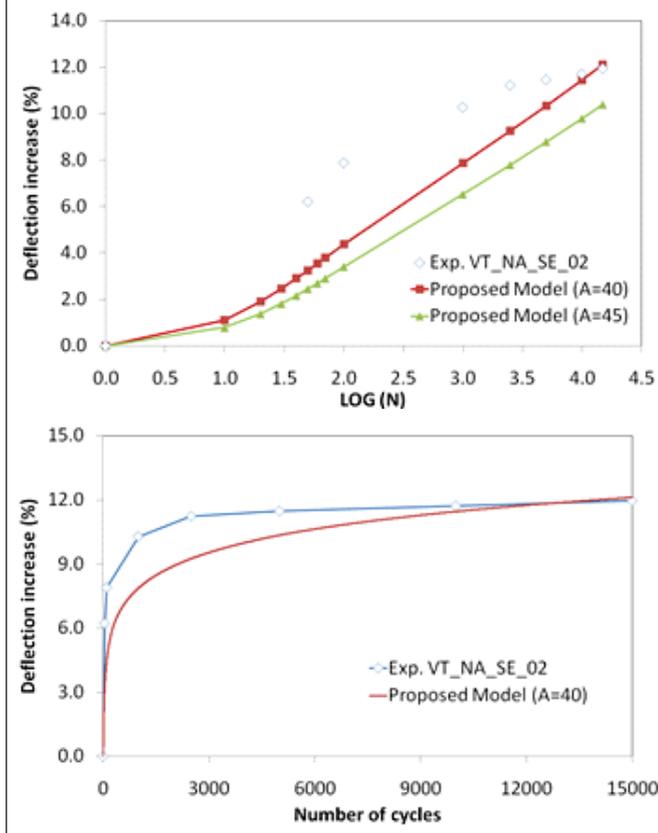
Table 6 contains the values in percentage of the increase in maximum deflections given by the expression presented at item 23.6 of NBR 6118 [19] and also calculated using the proposed model with  $k$  equal to 1000 to obtain the parameter  $A$  (Equation 6). The analysis of the results given in that table also shows the potential of the proposed model in simulating the effects of repeated cyclic loading. Obviously, additional experimental results on beams can contribute to a more precise evaluation of parameter  $A$  and also to assure the efficiency of the proposed expression to obtain  $z$ , parameter that controls the stiffness loss due to the cycles of loading.

## 5. Conclusions

In this work was presented a study on the structural behavior of reinforced concrete beams subjected to repeated cyclic loads. The main objective consisted in the evaluation of the loss of flexural stiffness by an experimental investigation and through a theoretical model. The main conclusions are as follows.



**Figure 16 – Theoretical and experimental results – Beam VT-NA-SE-02**



**5.1 About tests and experimental results**

Experimental program confirmed the effect of repeated cyclic loading on the mechanical deterioration of the beams, demonstrated by the increase in material strains (concrete and longitudinal reinforcement) and also by the growing deflections with loading cycles. The results also indicated the presence of residual strains and deflections caused by cycling (as figures 6 and 7).

The experimental results also indicated a strong influence of the

tensile reinforcement ratio on the cyclic behavior of the beams. In general, for beams with higher reinforcement rate (VR-AD-CE-01, VR-AD-CE-02 and VT-NA-SE-02), the increase of deflection related to the first cycle was lower than the observed in the beams with lower reinforcement rate (VR-NA-CE-01 and VR-NA-CE-02).

**5.2 About the theoretical model**

The nonlinear elastic model presented in this paper is capable to simulate the stiffness loss not only as a result of applied moments but also as a function of the nature of repeated cyclic loading.

A comparison with the experimental results obtained by Oliveira Filho [11] indicated that the theoretical model has potential to be used simulating the behavior of reinforced concrete beams submitted to repeated cyclic loading. For the prediction of maximum deflections, Equation (5) proposed in this work gave very satisfactory results.

One of the main limitations of the presented model is that it does not consider the cumulative residual strains of concrete and reinforcements caused by repeated loading- fact that was observed during testing by Oliveira Filho [11], even without the yielding of reinforcement.

To improve the proposed model, a future research will study a plasticity limit function that capture the inelastic rotations originated by residual strains of concrete and reinforcements- at first for monotonic loading and after for cyclic loading. Such investigation will start from the constitutive model proposed by Araújo [20]. This improvement of the theoretical model will permit that more conclusive analysis can be done about the proposed expression to obtain the parameter z.

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**Table 4 – Damage values at maximum bending moment sections obtained by the theoretical model**

Cycles	VR-NA-CE-01 (A=22)	VR-NA-CE-02 (A=17)	VR-AD-CE-01 (A=50)	VR-AD-CE-02 (A=40)	VT-NA-SE-02 (A=40)
1	0.62149	0.68410	0.40373	0.43875	0.48352
10	0.64757	0.70226	0.45078	0.48510	0.49646
100	0.68176	0.73876	0.50961	0.54699	0.53326
1000	0.70979	0.77240	0.54832	0.58913	0.56651
10000	0.73466	0.80140	0.58058	-	0.59592
15000	0.73911	0.80661	0.58604	-	0.60092

**Table 5 – Constant k: experimental values**

Beam	A	$\rho$	$\Delta P/P_{u,theor}$ (1)	$\Delta P/P_{u,exp}$ (2)	k (1)	k (2)
VR_NA_CE_01	22	0.00865	0.476	0.455	1211	1157
VR_NA_CE_02	17	0.00865	0.571	0.480	1452	943
VR_AD_CE_01	50	0.02182	0.481	0.445	1103	1021
VR_AD_CE_02	40	0.02182	0.519	0.477	952	875
VT_NA_SE_02	40	0.01510	0.385	0.347	1020	919

$P_{u,theor}$  = Predicted failure load     $P_{u,exp}$  = Experimental failure load

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**Table 6 – Percentual variation of maximum deflections: theoretical and experimental results**

Cycles	VR-NA-CE-01		VR-NA-CE-02		VR-AD-CE-01		VR-AD-CE-02		VT-NA-SE-02		NBR 6118 (c)
	(a)	(b)									
1	0	0	0	0	0	0	0	0	0	0	0
50	7.63	6.74	11.46	7.19	10.95	8.44	14.50	8.18	6.22	3.05	6.22
100	10.35	8.510	14.90	9.604	11.37	10.15	15.97	10.05	7.89	4.13	7.31
1000	14.17	14.69	20.92	18.97	13.68	15.09	18.49	15.62	10.29	7.61	12.26
2500	14.37	16.96	24.93	22.70	15.16	16.79	19.33	17.54	11.24	8.87	14.89
5000	15.80	18.77	26.65	25.64	15.58	18.11	19.54	19.04	11.48	9.88	17.16
10000	18.39	20.69	30.95	28.91	17.47	19.41	-	-	11.72	10.88	19.67
15000	20.71	21.87	31.23	30.86	18.32	20.25	-	-	11.96	11.50	21.25

(a): Experimental

(b): Proposed model-Parameter A obtained from Equation (6)

(c): Expression presented at item 23.6 of NBR 6118

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