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## Numerical and Experimental Study of a Real Scale Waffle Slab

# Análise Teórico-Experimental de uma Laje Nervurada em Escala Natural





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## Abstract

Nowadays, waffle slabs are a demand for structural designers, as a consequence of architectural design evolution and new building management concepts, in spite of its laborious numerical modeling. Therefore, it becomes necessary to know more about the structural behavior and improve the theoretical models used for simulating these slabs. The objective of this work was to analyze the adequacy of a design method widely used in the modeling of waffle slabs, verifying if it represents the slab behavior satisfactorily. A real scale waffle slab submitted to a load in a localized area was instrumented with strain gages and deflection gages for measuring specific strain and deflection in different points. The numerical analysis was made using a grid model developed by a local software company specialized in structural analysis. Tests showed a linear behavior, even though residual results could indicate cracking in some isolated sections. Numerically computed deflections presented a good estimate to test results and the experimental strains defined the presence of bending moments coincident with the forecasts of the theoretical model.

Keywords: waffle slab, reinforced concrete, grid analysis, instrumentation.

## Resumo

Soluções estruturais sofisticadas e racionais são exigências crescentes no cotidiano de projetistas de estruturas, como conseqüência da evolução dos projetos arquitetônicos e dos novos conceitos de gerenciamento das construções. As lajes nervuradas se enquadram nesta realidade como uma atraente alternativa, por propiciar economia de materiais e mão-de-obra, com redução de perdas e aumento da produtividade, exigindo, porém, uma laboriosa modelagem numérica. Para entender melhor como funciona, na prática, este sistema construtivo, torna-se necessário obter um maior conhecimento sobre seu comportamento estrutural, bem como aperfeiçoar os modelos teóricos empregados para seu projeto e simulação. O objetivo deste trabalho é analisar a adequação de métodos de cálculo empregados na modelagem destas estruturas, verificando se os mesmos representam satisfatoriamente seu comportamento. Para tanto, foi instrumentada uma laje nervurada de concreto armado em escala natural. O estudo mediu deformações no concreto e deslocamentos verticais em seções características da estrutura submetida a um carregamento localizado em uma área pré-estabelecida. A análise numérica foi feita empregando-se o modelo de análise matricial de grelhas do Sistema Computacional TQS v11.0. A análise dos resultados demonstrou que a laje nervurada em estudo apresentou uma tendência ao comportamento linear em todas as etapas de carregamento, sem fissuração da estrutura como um todo, embora os resíduos no processo de descarga tenham sugerido indícios de um início de fissuração em algumas seções isoladas. O comportamento da laje esteve dentro do previsto, com deslocamentos verticais na mesma ordem de grandeza das previsões teóricas e deformações específicas indicando a presença de momentos fletores nas seções instrumentadas coincidentes com os previstos pela análise numérica.

Palavras-chave: laje nervurada, concreto armado, análise matricial de grelhas, instrumentação.

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#### 1. Introduction

Structural analysis of building floors by computerized numerical methods is currently being used in structural design offices. The design with the aid of computer programs is almost essential, mainly because of the rate imposed by structural design contractors and the request to evaluate the various possibilities of structural systems, which must bring economic feasibility, speed and versatility of application (DIAS [1]).

In line with this trend, the use of waffle slabs is gradually becoming an attractive structural solution. This structural system can be defined as a grid of ribs, distributed in one or more directions, regularly spaced, connected by a top concrete slab (PEREIRA [2]).

The ribbed system is an evolution of solid slabs. It results from

the elimination of concrete below the neutral axis, which allows an economic increase on the total thickness of the slab with the creation of voids in a rhythmic arrangement. Therefore, there is a reduction on the structure self-weight and a more efficient use of materials, steel and concrete.

The static analysis of waffle slabs aims to determine the amount and distribution of shear forces, bending and torsional moments acting on the structure. It is intended to find out the required reinforcement necessary to satisfactorily resist to these efforts. Moreover, it is essential the evaluation of displacements that occur in the structure submitted to service load, considering a non-linear behavior.

Concerning to shear force, the greatest risk of failure comes from the punching of plate, which is characterized as a combined phenomenon of normal and tangential stresses. Thus, the area surrounding the columns, which must support punching efforts and





high negative bending moments, is reinforced with a solid slab region (MONTOYA [3]).

In Brazil, civil engineers still use a conventional reinforced concrete system (solid slab-beam-column). But, in recent years, there was a sudden increase in the use of waffle slabs. That however, makes necessary to develop procedures and standards to guide the use of such constructive system.

Knowledge of reinforced concrete ribbed slab behavior is yet unsatisfactory, although some numerical and experimental research results on this structural system can be found (Ajdukiewicz & Kliszczewicz 1986 [4]; Selistre 2000 [5]; Abdul-Wahab & Khalil 2000 [6]; Soares 2003 [7]; Schwetz 2005[8]).

Traditionally, waffle slabs have been analyzed using simplified methods, based on the Theory of Elasticity, developed for solid slabs. However, experimental tests confirm that this is not appropriate because the waffle slab geometry can not develop the same torsional moments of a solid slab with the same inertia. Therefore, it presents higher bending moments and vertical displacements.

Aiming to better understand the behavior of reinforced concrete ribbed slabs, and to quantify stresses and displacements of the structure in a more realistic way, a real scale waffle slab was submitted to a localized area load and instrumented for measuring specific strains and deflections. Results from a computational analysis were eventually compared to the experimental ones.

## 2. Numerical Analysis

Numerical methods and sophisticated computer programs are being used in the calculation of reinforced concrete structures. Among the available methods, the most used for the resolution of complex structures, such as waffle slabs, are those that analyze them as grids, following the procedures of Matrix Analysis or the Finite Element Method.

In this paper, the numerical analysis was performed using a matri-

cial grillage model developed by a Brazilian software house specialized in structural analysis, called *Sistema Computacional CAD/ TQS*. This software was chosen because it was the one adopted in the design of the structure under study, as indicated in item 2.1. (TQS[9]). The TQS system performs linear and non-linear analysis of the structure. In a linear analysis, the reinforcement is not considered in determining the stiffness of the cross-sections. The non-linear analysis calculates vertical displacements, considering the adopted reinforcement and the physical non-linearity of concrete due to cracking, based on standards of Brazilian Code NBR 6118:2003 (ABNT[10]).

#### 2.1 Slab Geometry and Load Distribution

The waffle slab under study was designed by a commercial design company of reinforced concrete structures, in the city of Porto Alegre, Brazil, that adopted the software CAD / TQS for Windows for the structural analysis. The modeling of the structure was made directly in the software's graphic interface and the designer was allowed to define his own design criteria.

This was the roofing floor of a commercial building and Fig 1 shows its final formwork and geometric properties. The whole structure consists of two towers, A and B, linked by an area where elevator and stairs are placed. The experimental program concerns only tower A, due to time and financial constraints.

To avoid excessive deformations, edge-beams were adopted by the structural designer. Also, the region surrounding the central pillar, which has negative bending moments and shear punching, was strengthened with a 32.5 cm depth solid slab region.

In the original design, the slab was exposed to its self-weight (4,1 kN/m<sup>2</sup>), to a permanent load (1,5 kN/m<sup>2</sup>) and to an accidental load (1,0 kN/m<sup>2</sup>). In the experimental program, however, the waffle slab was submitted only to a load in a localized area

In order to determine an adequate position and load intensity for this localized area, the structure was submitted to several numeri-



Table 1 - Numerically obtained verticaldisplacements within the loading area					
Vertical Displacements Results					
	linear (cm)	nonlinear (cm)			
Self-weight	0,38	0,38			
Total design load	0,62	0,67			
Localized area load, including self-weight	0,50	0,51			

cal analyses. Displacements and strains should be adequately measurable in the experimental tests. Furthermore, they should not be in excess of those resulting from the application of the original design load. With these limitations in mind, the numerical investigation defined as an appropriate area for the experimental load the one shown in Fig 2 and the load intensity adopted was 6.6 kN/m<sup>2</sup>.

Tables 1 and 2 show, respectively, numerically obtained displacement and bending moments, within the defined region, for the three loading situations: slab self-weight only, total design load and, selfweight plus the experimental load.

Fig 3 shows numerically obtained vertical displacements, within the experimental loading area, for a uniformly distributed incremental load applied on the whole slab. Also, Table 1, shows that the localized load (including self-weight) resulting displacement is 0.5 cm. Putting these information together, it can be determined, for the experimental load, an equivalent uniformly distributed load, applied in the whole slab, as far as displacements are concerned. This equivalent load corresponds to half of sum of permanent and accidental design loads. Such conclusion is shown in Fig 3, and Table 2 presents a similar idea regarding bending moments in the same region.

From the conclusion above, the defined experimental localized load was considered adequate for representing the waffle slab behavior in an experimental test.

Concrete was designed to achieve a compressive strength  $(f_{ck})$  of 30 MPa at 28 days and a modulus of elasticity (E) of 26 GPa, both specified in the original structure design. The clear cover was de-

## Table 2 - Numerically obtained bending moments within the loading area

Bendig Moments Results					
	linear (kNm)				
Self-weight	14,9				
Total design load	24,9				
Localized area load, including self-weight	19,9				

fined according to Brazilian Code NBR 6118:2003 (ABNT, 2004). The adopted Poisson's ratio was v = 0,2 and the value used for reinforced concrete weight per unit volume was  $\gamma_e = 25 kN/m^3$ . The software design criteria used in this work were the same set by the original structure designer, as defined below.

#### 2.2 CAD/TQS Main Design Criteria

TQS system offers the possibility of adopting some specific design criteria that allows the determination of efforts and reinforcement drawing details according to the usual practice of the structural designer. The main design criteria adopted in this work are listed below.

#### Restraint

The design criteria used for restraint in columns is called independent elastic restraint. In this case, each beam has a separate link to the column, defined by spring coefficients in the X and Y directions. These coefficients allow the reduction of clamping between beams and columns. The value adopted for the reduction coefficient was 4, which is the program default.

In addition to the springs in the X and Y directions, a spring in the Z direction may be introduced. The value adopted for reducing the spring coefficient in Z direction was 1, which is the program default.

#### Plastification on internal columns of waffle slabs

In intermediate columns, there are many ways to simulate plastification and to allow the consideration of torsion in the grid of the solid region around the columns. Thus, the bars inside the solid region are separated from the other ones, so that they could have more torsion and less flexure rigidity. The program extends the ribs into the solid region, completing the space between them with intermediate bars. The sum of the bar's width in each direction is equal to the solid region width, and the edge-bars have a half width. The bars inside the solid region have a torsion inertia divisor, while the edge-ribs may receive another divisor value. In this work, the torsion inertia divisor of the bars inside and outside of the solid region is the same and the value used was 6, which is the program default.

Moreover, all bars in the solid slab region have their flexure inertia reduced by a parameter, simulating plastification, which allows a better distribution of positive bending moments in the bars of the grid. This reduction parameter was taken as being equal to 1.6.

#### Torsion in grillage

The CAD/TQS system allows considering torsion in grillage ribs, although this is not the default criteria of the software. In this work, torsion was supported only by the columns and edge-beams.

#### 2.3 Numerical Results

After the necessary data has been provided, the software automatically generated a numerical model of the slab, consisting on 1335 nodes and 2276 bars, as shown in Fig 4. The brown points indicate the localized area load.

#### 2.3.1 Numerical Analysis for the localized area load, including the structure's self-weight

Fig 5 and 6 show, respectively, the deformed configuration of the slab and the distribution of bending moments for the application of



the localized area load, including self-weight. These results were obtained from a linear analysis of the waffle slab.

Performed a non-linear analysis, to verify maximum vertical displacements, it was obtained the results shown in Fig 7. The red lines represent bars in which the numerical analysis predicts cracking.

TQS system does not allow performing any analysis without considering the structure's self-weight. The experimental tests, however, measured vertical displacements only for the application of the localized area load, after the acting of the self weight. To compare such results, it was carried out an additional numerical analysis, considering only the slab self-weight. Thus, the desired numerical





results, concerning the effect of the localized area load only, could be obtained by the difference between both analyses.

#### 2.3.2 Numerical Analysis considering only waffle slab self-weight

Fig 8 and 9 show, respectively, the deformed configuration of the slab and the distribution of bending moments considering only the waffle slab self-weight. These results were obtained from a linear analysis of the slab.

A non-linear analysis was also performed and the results for vertical displacements are shown in Fig 10. It can be observed no prediction





of cracking in the grillage caused by self-weight. Interestingly, vertical displacements from the non-linear analysis result slightly lower then the ones obtained by linear analysis. This odd outcome resulted from the combination of two factors. The first is that self-weight did not cause slab cracking and, therefore, no loss of stiffness. The second factor is that the non-linear analysis accounts for the presence of the reinforcement, which the linear does not, thus considering a more rigid structure.

## 3. Experimental Program

The experimental program was conceived aiming the measuring of strains and vertical displacements in some specific sections of the





waffle slab, which was previously analyzed by a numerical model. The formwork was built with plastic cubes developed for waffle slabs by *Ulma Fôrmas e Escoramentos Ltda*. This system is composed by a modular supporting structure for the plastic cubes (Fig 11a), which is supported by metallic tubes that are very easily dismounted (Fig 11b). Reinforcement consisted of CA 50 and CA 60 steel bars, summarized in Tables 3 and 4. The general position of the rib reinforcement is found in Fig. 12. It can be observed that, in addition to positive and negative reinforcement, the designer specified a welded mesh, in CA 60 steel, for the top slab. This mesh was placed due to possible tension stresses in the top slab lower fibers, within the voids between the ribs.

Fig 13 shows the slab ready to be cast.

#### 3.1 Instrumentation

Concrete strain gauges were placed in three cross-section of the structure. In each of those points, there were strain gauges in the top and in the bottom face (Fig 14).

To measure vertical displacements, the waffle slab was instrumented with 5 deflection gauges. Those were fixed in concrete bases, leveled and positioned in the floor below (Fig 15). The position of instrumented points can be seen at Fig 16.

#### 3.2 Waffle Slab Test

The test was performed 46 days after the structure's casting and the load was applied in 4 steps, using cement bags, weighting 50 kg each (Fig 17).

At the end of each of the four loading and / or unloading steps, vertical displacements were measured by the deflection gauges, and specific strains obtained through a data acquisition system. Control specimen cylinders were cast along with the slab, to determine concrete's modulus of elasticity (E) and compressive strength ( $f_{ck}$ ). The modulus of elasticity was experimentally obtained by compressive force



Figure 11 – (a) Structure and plastic modular cubes and (b) Supporting system

according to the standards of Brazilian Code NBR 8522:1984 (ABNT [11]). The average value obtained for three specimens was E=35,74 GPa. The concrete compressive strength at 28 days was determined from the compression test of the control specimens, according to the recommendations of Brazilian Code NBR 5739:1994 (ABNT [12]). The value obtained for the concrete compressive strength was  $f_{rsc}$  = 37,2 MPa.

Table 3 – Summary of waffle slab positive reinforcement					
Steel	Diam. (mm)	Lenght (m)	Weight (kg)		
CA 50	6,3	67	17		
CA 50	10	836	527		
CA 50	12.5	1280	1280		
CA 50	16	169	271		
CA 50	20	24	60		
Total weight CA 50: 2154 kg					

Table 4 – Summary of waffle slab negative reinforcement					
Steel	Diam. (mm)	Lenght (m)	Weight (kg)		
CA 60	5	33	5		
CA 50	6.3	739	185		
CA 50	10	670	422		
CA 50	12.5	339	339		
CA 50	16	103	164		
CA 50	20	22	55		
Total weight CA 60: 5 kg; Total weight CA 50: 1165 kg					

# 4. Comparison between Test and Numerical Results

The tested slab was numerically analyzed considering two sets of values for the parameters compressive strength ( $f_{ck}$ ) and modulus of elasticity (E), the ones specified by the structural designer and the obtained experimentally from the test specimens.

Numerical and experimental results were then compared, concerning specific strains and vertical displacements, in some specific points showed in Fig 16.

#### 4.1 Vertical Displacements

Figures 18 to 22 show experimentally measured vertical displacements, compared to the numerical ones, obtained from the slab non-linear analyses considering both values for compressive strength ( $f_{ev}$ ) and modulus of elasticity (E).

The slab presented a near linear behavior under crescent loading and test results were very close to the numerical ones. Instrumented points 1 and 2, showed displacements close to the numerical predictions when adopting the experimental modulus of elasticity. On the other hand, points 3, 4 and 5, agreed more closely with numerical results adopting the modulus specified by the designer.



Figure 13 - Slab before casting



It is noticed that deflection gauges 3, 4 and 5 presented some residual results during unloading, which may indicate a beginning of cracking in the area after total loading, However, the slab did not show any visual signs of cracking.

#### 4.2 Specific Strain

The numerical values for specific strains were obtained from the bending moment results. For such calculation, the section moments of inertia were determined considering the presence of the reinforcement, for both cracked and uncracked cross-section, applying the Transformed Cross-Section Method (PARK [13])

Table 5 shows the calculated values of strains, in concrete and at the reinforcement, for cracked and uncracked sections and for both modulus of elasticity, determined according to recommendations of Brazilian Code NBR 6118:2003 (ABNT [6])

Figs 23 to 28 present the comparison between numerical and experimental strains, for each load stage in all instrumented points.



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TQS system models a waffle slabs as side-by-side "T" cross-section beams. Therefore, there are no bending moment results for the top slab, within the voids between the ribs. For such reason, Figs 23 and 24 show only experimental results for instrumented point 3. These data shall be compared to results from a future finite element analysis. General results made possible to observe that the waffle slab





developed tension in the bottom fibers and compression in the top ones, indicating the occurrence of positive bending moments as numerically predicted. Also, that the slab had a linear elastic path along the loading test, suggesting an over-











all uncracked behavior, in accordance with the results of the deflection gauges.

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Nonetheless, some strain gauges show residual results for the structure unloading. That might indicate areas where cracking was

Table 5 – Specific strain for cracked and uncracked sections with modulus of elasticity E= 35,74 GPa and E= 26 GPa										
logd Orea	Cross	Bending Moment (kNm)	Uncracked section			Cracked section				
Step	Section		ε <sub>sup</sub> (x10 <sup>-6</sup> ) (E=26 GPa)	ε <sub>inf</sub> (x10 <sup>-6</sup> ) (E=26 GPa)	ε <sub>sup</sub> (x10 <sup>-6</sup> ) (E=35,74 GPα)	ε <sub>inf</sub> (x10 <sup>-</sup> ) (E=35,74 GPa)	ε <sub>sup</sub> (x10 <sup>-6</sup> ) (E=26 GPa)	ε <sub>inf</sub> (x10 <sup>-6</sup> ) (E=26 GPa)	ε <sub>sup</sub> (x10 <sup>-6</sup> ) (E=35,74 GPα)	ε <sub>inf</sub> (x10 <sup>-</sup> ) (E=35,74 GPa)
1	1	1,38	-5,89	11,96	-4,35	9,03	-10,24	88,1	-8,57	86,83
	2	1,43	-6,11	12,39	-4,51	9,36	-10,45	68,31	-8,75	67,66
2	1	2,75	-11,79	23,92	-8,71	18,07	-20,48	176,19	-17,15	173,65
	2	2,85	-12,22	24,79	-9,02	18,72	-20,89	136,62	-17,5	135,32
3	1	4,13	-17,68	35,88	-13,06	27,1	-30,71	264,29	-25,72	260,48
	2	4,28	-18,33	37,18	-13,54	28,08	-31,34	204,93	-26,25	202,98
4	1	5,50	-23,58	47,84	-17,41	36,13	-40,95	352,38	-34,3	347,31
	2	5,70	-24,44	49,58	-18,05	37,44	-41,79	273,24	-35	270,64











just developing under the total load, but not yet imposing a cracking like behavior for the whole structure. That was also indicated by the numerical analysis.

Figs 23 and 24 indicate the presence of bending stresses in top slab, within the voids between ribs, thus confirming the adequacy of the welded steel mesh specified by the design engineer.

## 5. Final Conclusions

The presented results showed that the waffle slab under study behaved in a linear like fashion during the loading process, indicating no overall cracking, but only in some localized areas, as numerically predicted.

It was also observed that the measured vertical displacements and strains were satisfactorily close to the numerically predicted ones, indicating an adequate response of the numerical model used in the analysis.

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